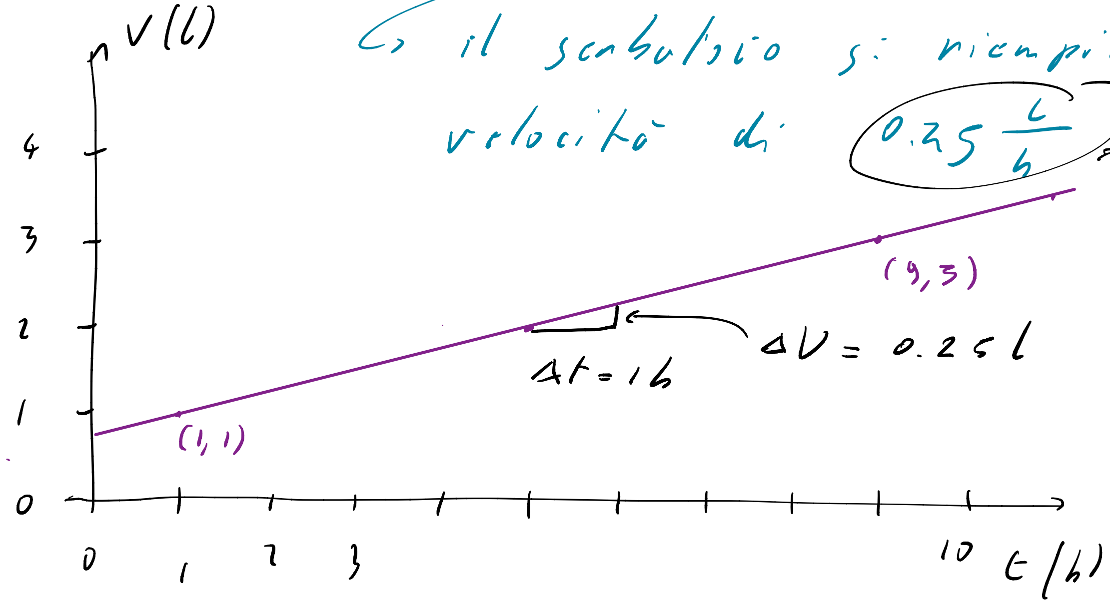
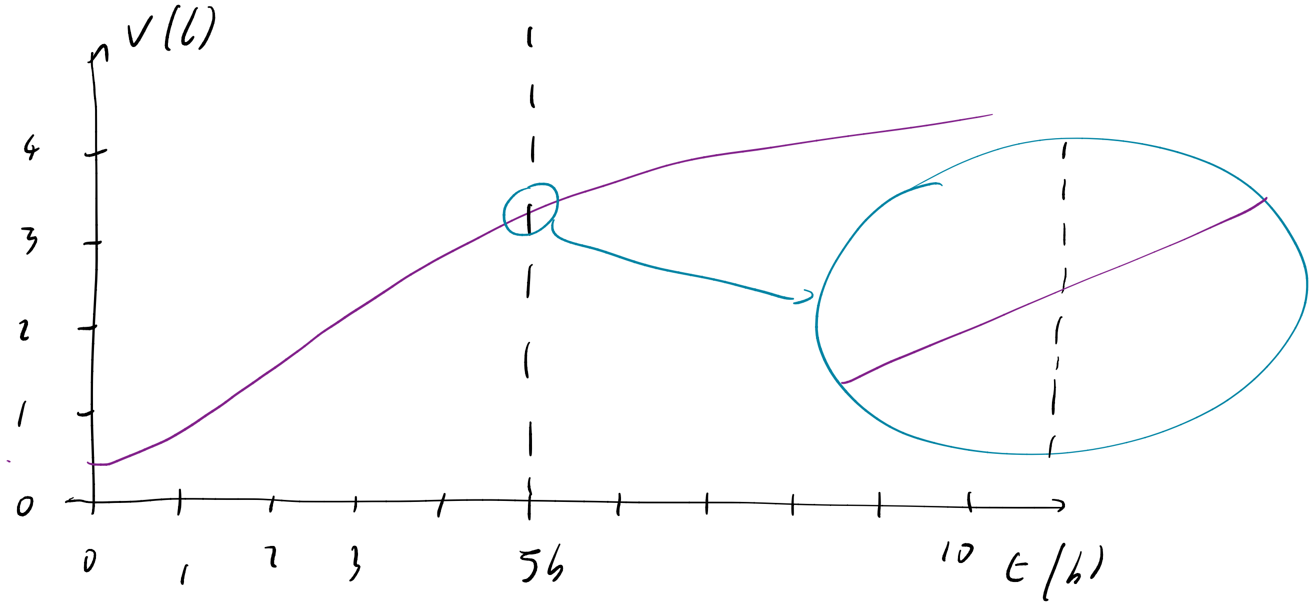
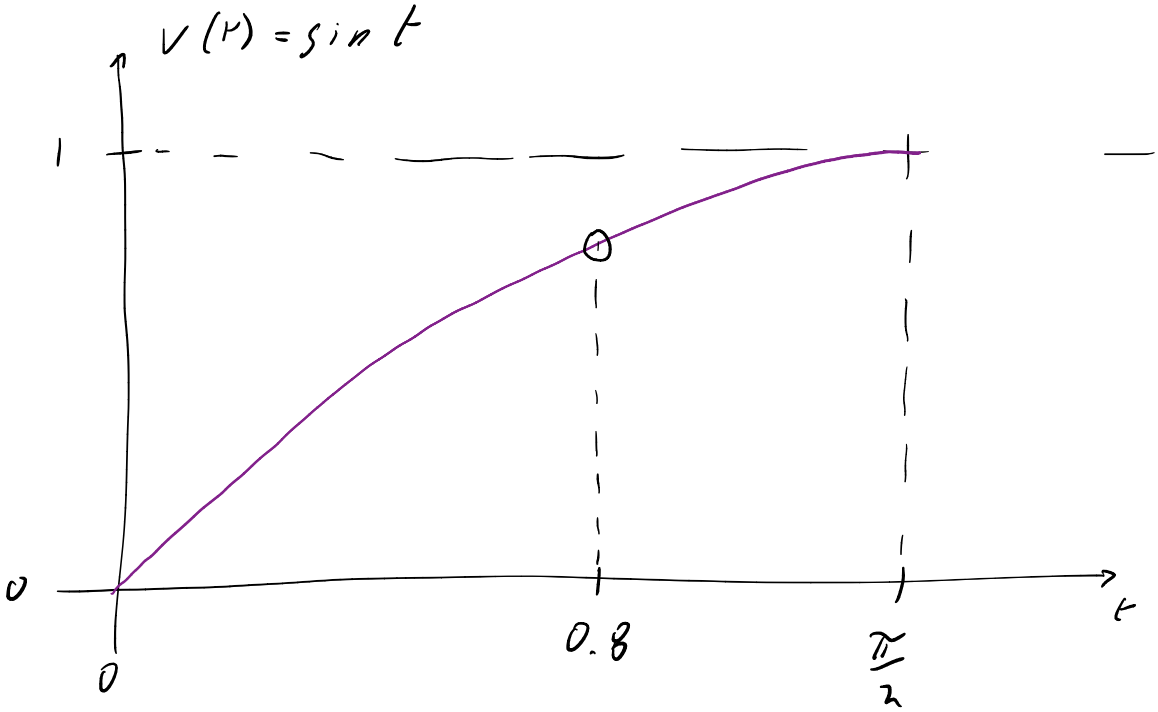
La derivata

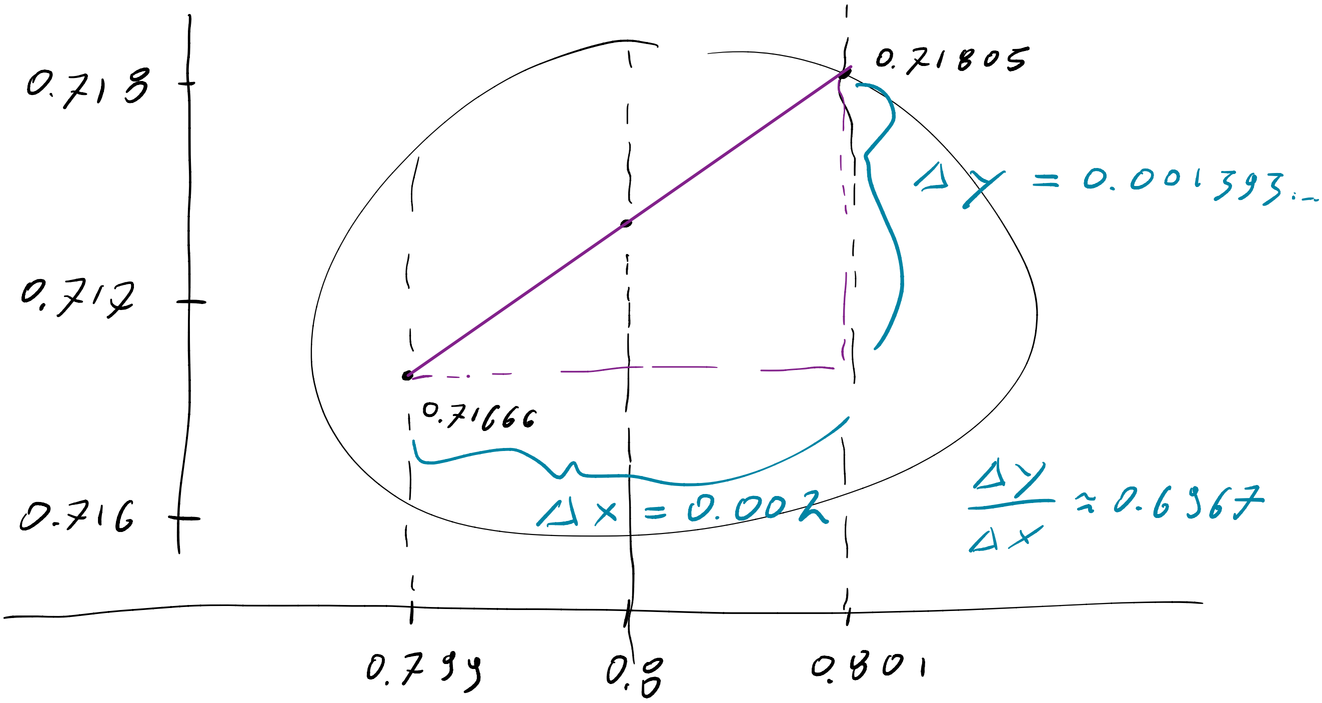
$$V = 0.75\,l + t \cdot 0.25\,\frac{l}{h}$$

↳ il serbatoio si riempie alla velocità di $0.25\,\frac{l}{h}$ $\frac{\Delta V}{\Delta t}$

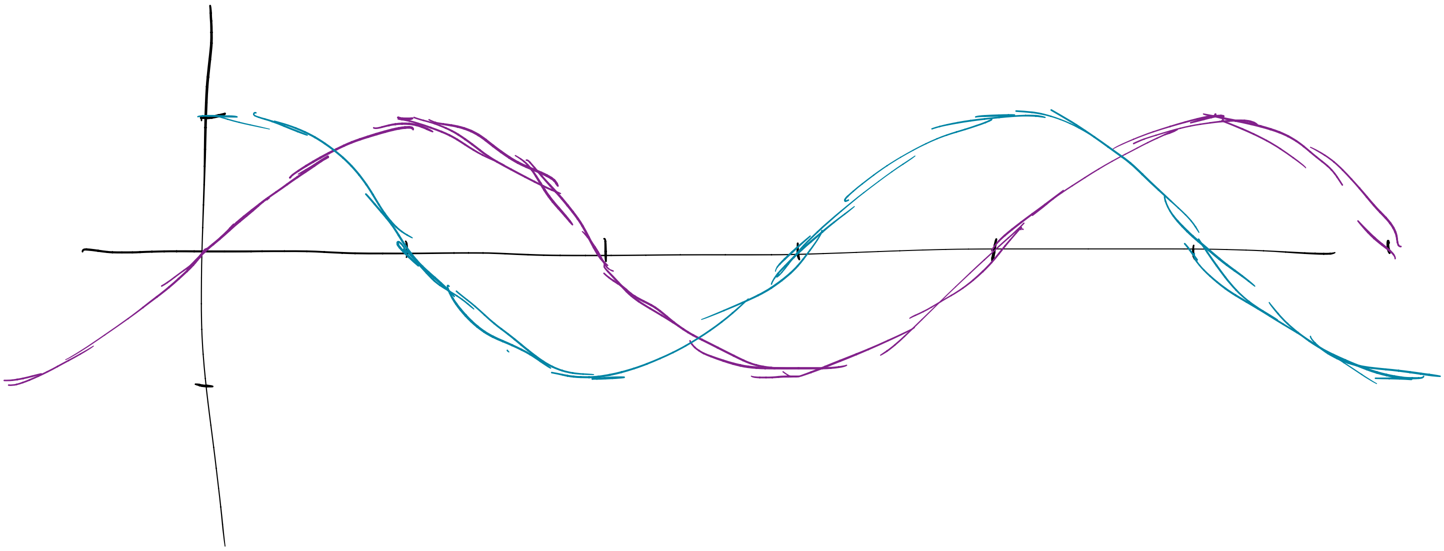








$\cos 0.8 \approx 0.6967\ldots$



$f'(x)$ = la derivata di $f(x)$

$\sin'(x) = \cos(x)$

Derivate delle f. elementari (p. 134)

$(\text{costante})' = 0$ $\qquad$ $(ax+b)' = a$

$(\sin x)' = \cos x$ $\qquad$ $(\cos x)' = -\sin x$

$(e^x)' = e^x$ $\qquad$ $(\log_e x)' = \dfrac{1}{x}$

$(x^a)' = a \cdot x^{a-1}$

$(\arcsin x)' = \dfrac{1}{\sqrt{1-x^2}}$

$(\arctan x)' = \dfrac{1}{1+x^2}$

$(\arccos x)' = \dfrac{-1}{\sqrt{1-x^2}}$

Regole

$$\left(f(x) \pm g(x)\right)' = f'(x) \pm g'(x)$$

$$\left(f(x) \cdot g(x)\right)' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

$$\left(k\, f(x)\right)' = k\, f'(x)$$

$$\left(f(g(x))\right)' = f'(g(x)) \cdot g'(x)$$

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

$$\left(\frac{f(x)}{g(x)}\right)' = \left(f(x) \cdot \frac{1}{g(x)}\right)' = \left(f(x) \cdot (g(x))^{-1}\right)'$$

$$= f'(x) \cdot (g(x))^{-1} + f(x) \cdot \underline{\left((g(x))^{-1}\right)'} = (*)$$

$$x \xmapsto[]{\substack{g'(x) \\ g(x)}} g(x) \xmapsto[]{\substack{f'(x) = -x^{-2} \\ f(x) = x^{-1}}} (g(x))^{-1}$$

$$\left((g(x))^{-1}\right)' = -(g(x))^{-2} \cdot g'(x) \qquad \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

$$\|$$

$$(*) = f'(x) \cdot (g(x))^{-1} - f(x) \cdot g'(x) \cdot (g(x))^{-2}$$

$$\left((x+1)^2\right)' = \left(x^2+2x+1\right)'$$

$$= \left(x^2\right)' + \left(2x\right)' + \left(1\right)'$$

$$= 2x + 2 + 0 = 2x+2$$

$$x \xmapsto[\quad]{\substack{g'(x)=1 \\ g(x)=x+1}} x+1 \xmapsto[\quad]{\substack{f'(x)=2x \\ f(x)=x^2}} (x+1)^2$$

$$\left(f(g(x))\right)' = f'(g(x)) \cdot g'(x)$$

$$= 2(x+1)\cdot 1 = 2x+2$$

$$\left(\sin(x^2)\right)'$$

$$x \xmapsto[\quad]{\substack{g'(x)=2x \\ g(x)=x^2}} x^2 \xmapsto[\quad]{\substack{f'(x)=\cos(x) \\ f(x)=\sin(x)}} \sin(x^2)$$

$$\left(f(g(x))\right)' = f'(g(x)) \cdot g'(x)$$

$$= \cos(x^2)\cdot 2x = 2x\cos(x^2)$$

$$\left(\sin(x^2)\cdot\cos(x)\right)' =$$

$$= \left(\sin(x^2)\right)'\cdot\cos(x) + \sin(x^2)\cdot\left(\cos(x)\right)'$$

$$= 2x\cos(x^2)\,\cos(x) + \sin(x^2)\left(-\sin(x)\right)$$

$$= 2x\cos(x)\cos(x^2) - \sin(x)\sin(x^2)$$

$$\left(\tan(x)\right)' = \left(\frac{\sin(x)}{\cos(x)}\right)'$$

$$= \frac{(\cos(x))^2 + (\sin(x))^2}{(\cos(x))^2} = \frac{1}{(\cos(x))^2}$$

$$\left(7^x\right)' = \left(\left(e^{\log_e 7}\right)^x\right)' = \left(e^{(\log_e 7)\cdot x}\right)' = \circledast$$

$$x \xmapsto{\substack{g'(x) = \log_e 7 \\ g(x) = (\log_e 7)\cdot x}} (\log_e 7)\cdot x \xmapsto{\substack{f'(x) = e^x \\ f(x) = e^x}} e^{(\log_e 7)\cdot x}$$

$$\circledast = e^{(\log_e 7)\cdot x}\cdot\log_e 7 = 7^x\cdot\log_e 7$$

$$(k^x)' = k^x \cdot \log_e k$$

$$\left(f(ax+b)\right)' = a f'(ax+b)$$

$$(\log_{10} x)' = \left(\frac{\log_e x}{\log_e 10}\right)' = \left(\frac{1}{\log_e 10} \cdot \log_e x\right)'$$

$$= \frac{1}{x \log_e 10}$$

$\boxed{\text{Es. dai test: p. 136-139}}$

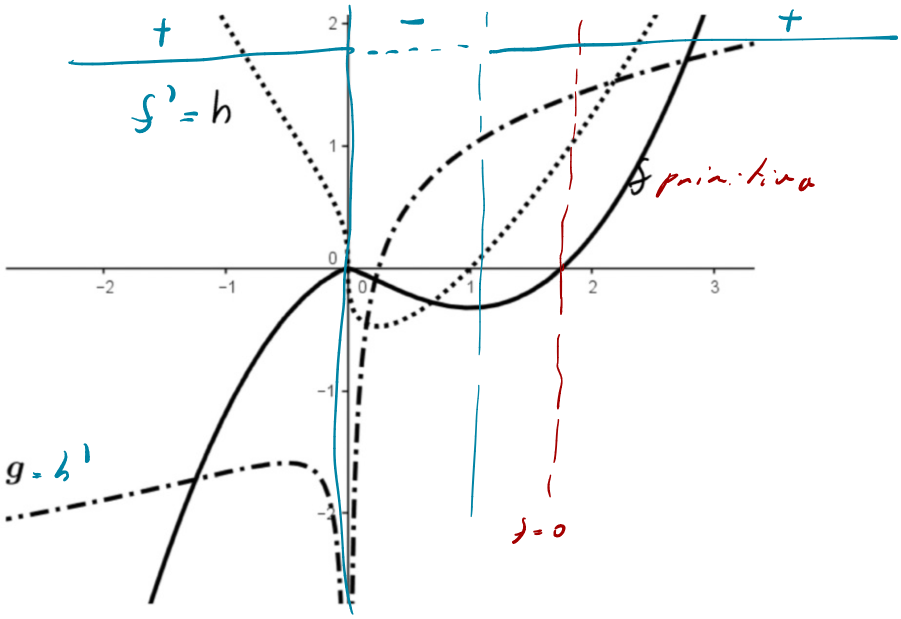
$$(x^x)' = \left(\left(e^{\log_e x}\right)^x\right)' = \left(e^{x \cdot \log_e x}\right)' = (*)$$

$g'(x) = \log_e x + 1$ $\qquad$ $f'(x) = e^x$

$$x \xmapsto{g(x) = x \cdot \log_e x} x \cdot \log_e x \xmapsto{f(x) = e^x} e^{x \log_e x}$$

$$(*) = e^{x \cdot \log_e x} \cdot (\log_e x + 1) = x^x (1 + \log_e x)$$

Esercizio (dal corso della prof. Baccaglini-Frank)
Il grafico seguente rappresenta tre funzioni di cui una è la primitiva, una la derivata, e una la derivata seconda (derivata della derivata). Chi è chi?



Fare es. a p. 200