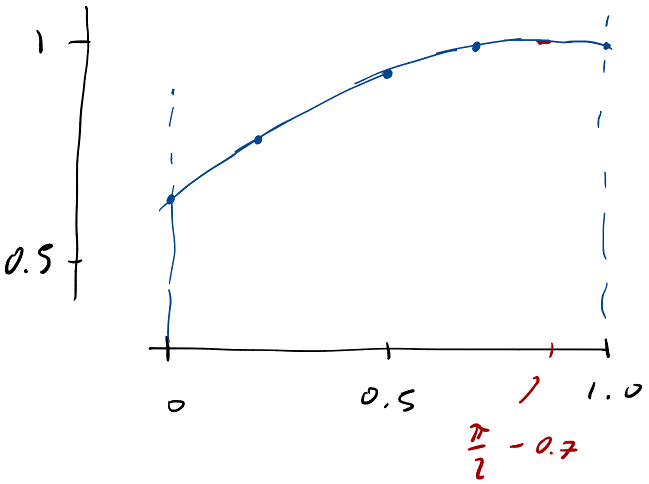
Esercizio D

Determina il massimo ed il minimo dei valori assunti da $f(x) = \sin(x + 0.7)$ al variare di x nell'intervallo $[0, 1]$.

$\frac{\pi}{2} - 0.7 \simeq 0.87$

x	0	0.2	0.5	0.7	$\square$	1
$f(x)$	0.64	0.78	0.93	0.99	1	0.99



max val. = 1

min = $\sin(0.7)$

$$(f(ax+b))' = a f'(ax+b)$$

$$f(x) = \sin(x + 0.7)$$

$$f'(x) = \underbrace{\cos(x + 0.7) = 0}$$

$$x + 0.7 = \frac{\pi}{2} + k\pi \qquad k \in \mathbb{Z}$$

$$x = \frac{\pi}{2} - 0.7 + k\pi$$

già visto $\frac{\pi}{2} - 0.7$

$\simeq 0.87$

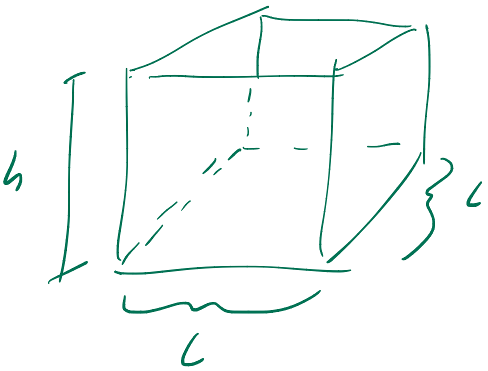
k	-1	0	1
	~ -2.68	$\boxed{\sim 0.87}$	~ 4.01

x	0	$\frac{\pi}{2} - 0.7$	1
$f(x)$	$\sin(0.7) \simeq 0.64$ MIN	1 MAX	$\sin(1.7) \simeq 0.99$

$\uparrow$ $\sin(x+0.7)$

Esercizio B

Si vuole costruire una scatola di cartone a forma di parallelepipedo a base quadrata, chiusa sotto ma aperta sopra, del volume di 10 litri. Determina il lato di base l e l'altezza h del parallelepipedo in modo tale che la superficie di cartone impiegata sia minima possibile.



$$V = l^2 \cdot h \quad (10\ dm^3)$$

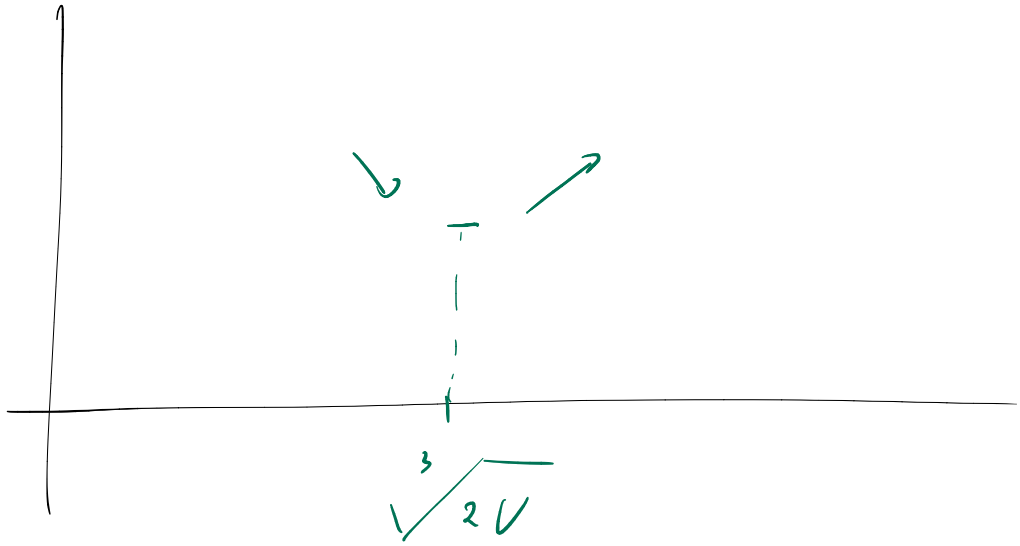
$$S = l^2 + 4lh$$

$$h(l) = \frac{V}{l^2}$$

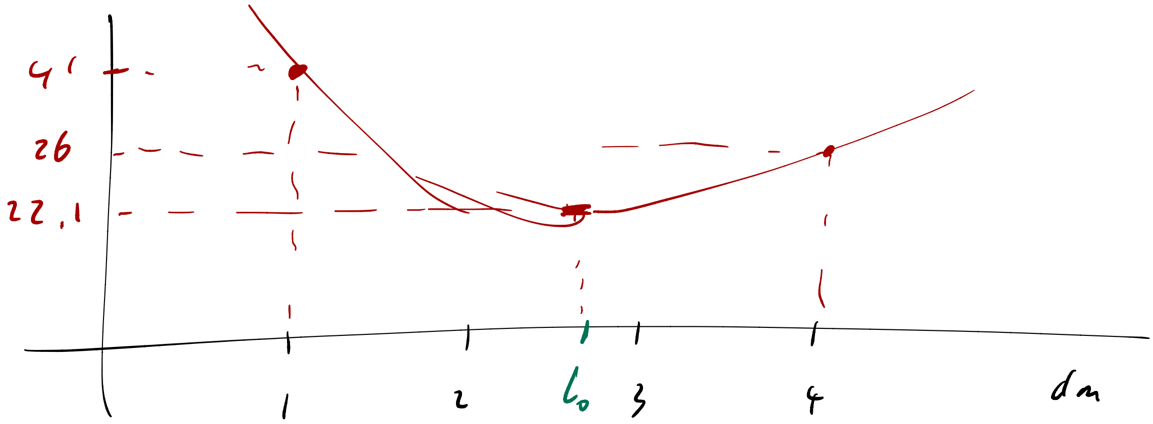
$$S(l) = l^2 + 4l\frac{V}{l^2} = l^2 + 4\frac{V}{l}$$

$$S'(l) = \underbrace{2l - 4V\frac{1}{l^2} = 0}$$

$$2l = 4V\frac{1}{l^2} \qquad l = \sqrt[3]{2V}$$



$$S(l) = l^2 + \frac{4V}{l} = l^2 + \frac{40\ dm^3}{l} \qquad l_0 = \sqrt[3]{2V} \approx 2.71\ dm$$



Il lato corretto è $l_0 = \sqrt[3]{2V} \approx 2.71\ dm$

$h = \ldots$

Esercizio A

Determina il massimo ed il minimo dei valori assunti dalla funzione

$$f(x) = x - \sin(x) - \cos(x)$$

al variare di x nell'intervallo $[-2, 2]$. Traccia quindi un grafico di questa funzione con il medesimo intervallo in ascissa.

$$f'(x) = \underbrace{1 - \cos(x) + \sin(x)}\ = 0$$

$$\cos(x) - \sin(x) = 1$$

$$\pm\sqrt{1-(\cos(x))^2} = \cos(x) - 1$$

$$1 - (\cos(x))^2 = (\cos(x))^2 + 1 - 2\cos(x)$$

$$2(\cos(x))^2 - 2\cos(x) = 0$$

$$\cos(x) = 0 \qquad\qquad \cos(x) = 1$$

$$x = \frac{\pi}{2} + k\pi \qquad\qquad x = 2k\pi \quad ✓$$

$$x = \frac{\pi}{2} + 2k\pi \quad ✗$$

$$x = \frac{3}{2}\pi + 2k\pi \quad ✓$$

k	.	-1	$\boxed{0}$	1	...
x	...	-6.28	$\boxed{0}$	6.28	...

k	$\boxed{-1}$	0
x	$\boxed{-1.57}$	4.71

$$-\frac{\pi}{2}$$

$$\cos(x) - \sin(x) = 1 \qquad \alpha = \frac{\pi}{4}$$

$$\underbrace{\overset{\frac{\sqrt{2}}{2}}{\sin(\alpha)}\cos(x) - \overset{\frac{\sqrt{2}}{2}}{\cos(\alpha)}\cdot\sin(x)}_{\sin(\alpha - x)} = \frac{\sqrt{2}}{2}$$

$$\sin\left(\frac{\pi}{4} - x\right) = \frac{\sqrt{2}}{2}$$

$$\frac{\pi}{4} - x = \begin{cases} \overset{\frac{\pi}{4}}{\arcsin\left(\frac{\sqrt{2}}{2}\right)} + 2k\pi \\ \pi - \overset{\frac{\pi}{4}}{\arcsin\left(\frac{\sqrt{2}}{2}\right)} + 2k\pi \end{cases}$$

$$\frac{\pi}{4} - x = \frac{\pi}{4} + 2k\pi \qquad x = -2k\pi$$

$$\frac{\pi}{4} - x = \pi - \frac{\pi}{4} + 2k\pi \qquad x = -\frac{\pi}{2} - 2\pi\pi$$

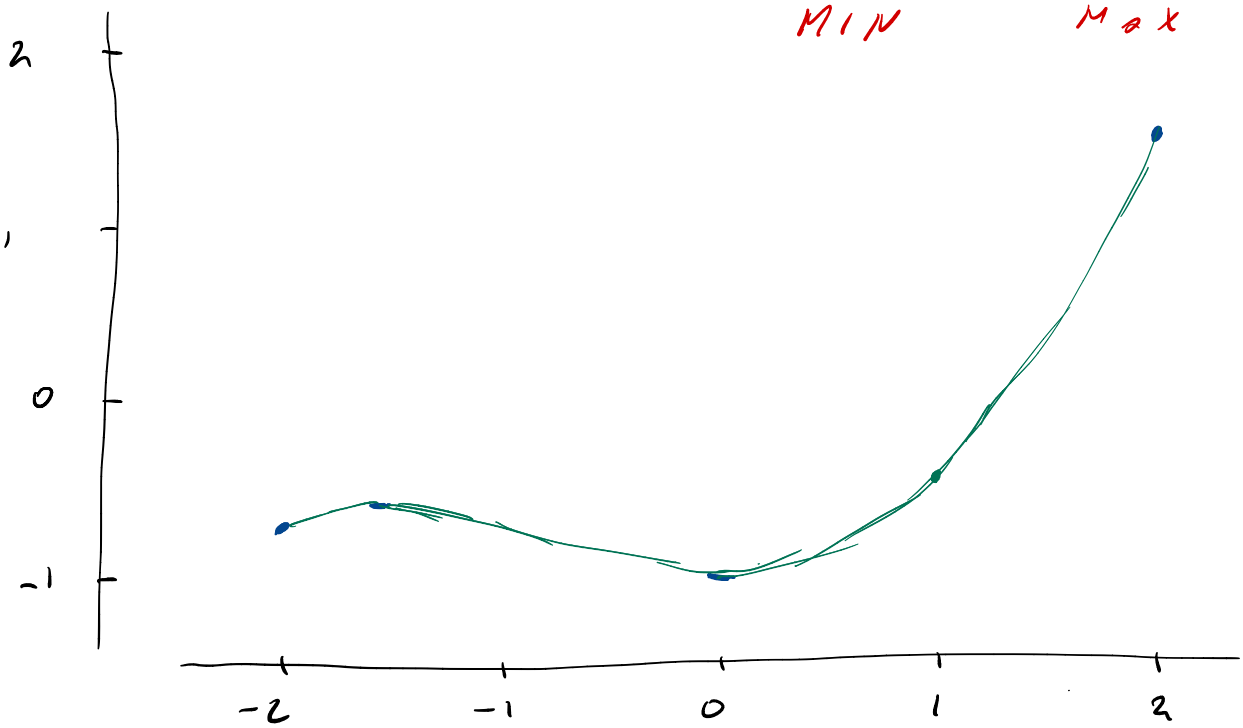
$$\boxed{A\sin(x) + B\cos(x) = C}$$

$$\overset{\cos(\alpha)}{\frac{A}{\sqrt{A^2+B^2}}}\sin(x) + \overset{\sin(\alpha)}{\frac{B}{\sqrt{A^2+B^2}}}\cos(x) = \frac{C}{\sqrt{A^2+B^2}}$$

$f(x) = x - \sin(x) - \cos(x)$

p.ti staz. $x = 0$ $x = -\frac{\pi}{2}$

x	-2	$-\frac{\pi}{2}$	0	2
$f(x)$	-0.67	-0.57	$\underline{-1}$	$\underline{1.51}$
			MIN	MAX



c) Trova un polinomio di terzo grado $f(x)$ tale che $\boxed{x^2 + x \cdot f'(x) = 3 \cdot f(x)}$ e $\boxed{f(1) = 0}$.

$$f(x) = ax^3 + bx^2 + cx + d$$

$$f'(x) = 3ax^2 + 2bx + c$$

$$x^2 + x(3ax^2 + 2bx + c) = 3(ax^3 + bx^2 + cx + d)$$

$$3ax^3 + x^2(2b+1) + cx + 0 = 3ax^3 + 3bx^2 + 3cx + 3d$$

$$3a = 3a \longrightarrow a = ?$$

$$\left.\begin{array}{l} 2b+1 = 3b \\ c = 3c \\ 0 = 3d \end{array}\right\} \quad \begin{array}{l} b = 1 \\ c = 0 \\ d = 0 \end{array}$$

$$f(x) = ax^3 + x^2$$

$$f(1) = 0$$

$$a \cdot 1 + 1 = 0 \iff a = -1$$

$$\boxed{f(x) = -x^3 + x^2}$$