Eq. differenziali

Es. $x^2 + x f'(x) = 3 f(x)$

$\Downarrow$

$$f(x) = C x^3 + x^2$$

Verifica $f'(x) = 3Cx^2 + 2x$

$$\underbrace{x^2 + x\left(3Cx^2 + 2x\right)}_{3Cx^3 + 3x^2} = \underbrace{3\left(Cx^3 + x^2\right)}_{3Cx^3 + 3x^2}$$

$3Cx^3 + 3x^2 \;=\; 3Cx^3 + 3x^2$

ok ✓

ordine di una eq. diff. = massimo ordine di derivazione di f.

$\Downarrow$

soluzione = famiglia di funzioni a ORDINE parametri

"condiz. iniziale"

Es $f(1) = 0$ $\qquad$ $f(x) = Cx^3 + x^2$

$0 = C \cdot 1 + 1$

$\to C = -1$

$f(x) = -x^3 + x^2 \Leftarrow$

Integrali indefiniti

$f'(x) =$ espressione data

Es. $f'(x) = 2x \longleftarrow \int 2x \, dx$

$\Downarrow \qquad\qquad \| $

$f(x) = x^2 + C \qquad x^2 + C$

$$\int x^{\alpha} dx = \frac{1}{\alpha+1} x^{\alpha+1} + C \quad \text{per } \alpha \neq -1$$

$$\int \frac{1}{x} dx = \log_e x + C$$

$$\int \sin x \, dx = -\cos x + C \qquad \int \cos x \, dx = \sin x + C$$

$$\int e^x dx = e^x + C$$

$$\int f(x)\,dx = F(x) + C$$

$$\int g(x)\,dx = G(x) + C$$

$\Leftarrow$ fissiamo.

$$\int k \cdot f(x)\,dx = k\,F(x) + C$$

$$\int 4 \underbrace{\sin(x)}_{f}\,dx = -4\cos(x) + C$$

$F(x) = -\cos(x)$

$$\int f(x) + g(x)\,dx = F(x) + G(x) + C$$

$$\int \sin(x) + \cos(x)\,dx = -\cos(x) + \sin(x) + C$$

$$\int f(g(x))\,g'(x)\,dx = F(g(x)) + C$$

$1 = \frac{1}{2} \cdot 2$

$$\int x \cdot \sin(x^2)\,dx =$$

$f(x) = \sin x$

$g(x) = x^2$

$$= \int \frac{1}{2} \cdot 2x \cdot \sin(x^2)\,dx = \frac{1}{2}\int 2x \cdot \sin(x^2)\,dx$$

$$= \frac{1}{2}\left(-\cos(x^2)\right) + C$$

$$\int \sin(x) \cdot \sqrt{\cos(x)}\, dx$$

$F(x) = \frac{1}{\frac{3}{2}} x^{\frac{3}{2}}$

$\uparrow$

$f(x) = \sqrt{x}$

$$= -\int -\sin(x)\sqrt{\cos(x)}\, dx$$

$g(x) = \cos(x)$

$$= -\frac{2}{3}\left(\cos(x)\right)^{\frac{3}{2}} + C$$

$$\int \sin x \cdot \cos x \, dx = \frac{1}{2}(\sin x)^2 + C$$

$f(x) = x \qquad F(x) = \frac{1}{2}x^2$

$g(x) = \sin x$

$$\sin(2x) = 2\sin x \cos x$$

$$\sin x \cos x = \frac{1}{2}\sin(2x)$$

$$* = \frac{1}{2 \cdot 2}\int 2\sin(2x)\, dx = \frac{1}{4}\left(-\cos(2x)\right) + C$$

$f(x) = \sin(x) \qquad g(x) = 2x$

$$-\frac{1}{4}\cos(2x) + C = \frac{1}{2}(\sin x)^2 - \frac{1}{4} + C$$

$$\frac{1}{2}(\sin x)^2 + C$$

$$\cos(2x) = 1 - 2(\sin x)^2$$

$$\int f(ax+b)\,dx = \frac{1}{a}F(ax+b) + C$$

$$\int e^x\,dx = e^x + C$$

$$\int 10^x\,dx = \int e^{x\cdot\log_e 10}\,dx$$

$$= \frac{1}{\log_e 10}\,e^{x\cdot\log_e 10} + C = \frac{1}{\log_e 10}\,10^x + C$$

$$\int a^x\,dx = \frac{1}{\log_e a}\cdot a^x + C$$

p. 232 – 237 $\quad B_5 \quad$ p. 238