Equazioni a variabili separabili p. 249

$$f'(x) = g(x) \cdot h\left(f(x)\right)$$

$$\frac{df}{dx} = g(x) \cdot h(f)$$

$$\int \frac{df}{h(f)} = \int g(x)\, dx$$

Es. $f' = 3xf$

$$\frac{df}{dx} = 3xf \qquad \int \frac{df}{f} = \int 3x\, dx$$

$$\log_e f + C_1 = \frac{3}{2}x^2 + C_2$$

$$\log_e f = \frac{3}{2}x^2 + \underbrace{C_2 - C_1}_{C}$$

$$f(x) = e^{\frac{3}{2}x^2 + C} = e^C e^{\frac{3}{2}x^2} = \underline{K e^{\frac{3}{2}x^2}}$$

$$f' = 3x f \qquad f(x) = k\, e^{\frac{3}{2}x^2}$$

$$f'(x) = k\, e^{\frac{3}{2}x^2} \cdot \frac{3}{\cancel{2}} \cdot \cancel{2}x$$

$$\Downarrow$$

$$k\, e^{\frac{3}{2}x^2} \cdot 3x \overset{\checkmark}{=} 3x\, k\, e^{\frac{3}{2}x^2}$$

$$x'(t) = \frac{3x(t)}{t} \qquad f'(x) = \frac{3 f(x)}{x}$$

$$\frac{dx}{dt} = \frac{3x}{t}$$

$$\int \frac{dx}{x} = \int \frac{3}{t}\, dt$$

$$\log_e x = 3 \log_e t + C \qquad x' = k\, 3 t^2$$

$$\Uparrow$$

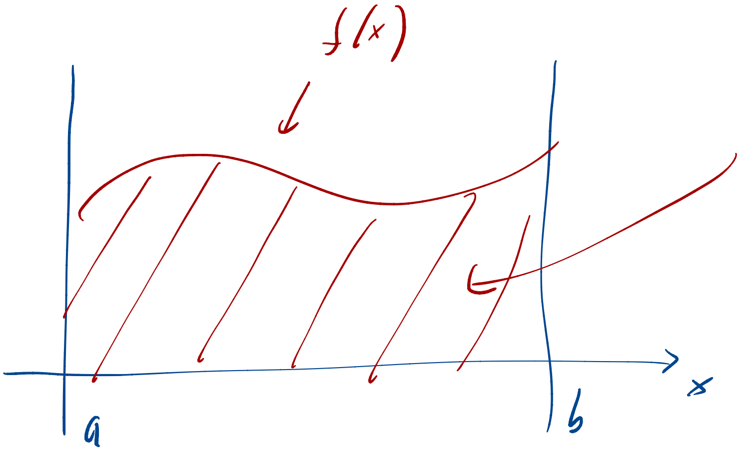
$$\boxed{x(t)} = e^{C + 3\log_e t} = e^C \cdot t^3 = \boxed{k\, t^3}$$

$$x'(t) = \frac{3x(t)}{t} \qquad k\, 3 t^2 \overset{\checkmark}{=} \frac{3 k t^3}{t}$$

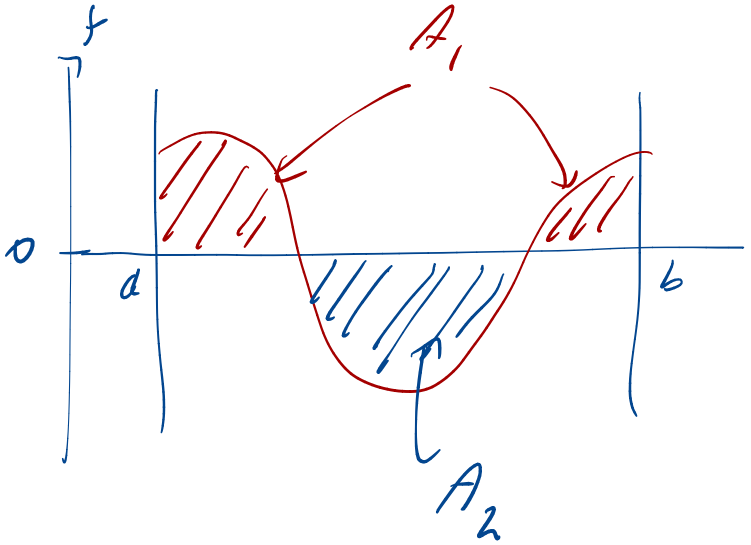
Integrali definiti

$$\int f(x)\,dx = F(x) + C$$

$$\int_a^b f(x)\,dx = F(b) - F(a) = \left[F(x)\right]_a^b$$



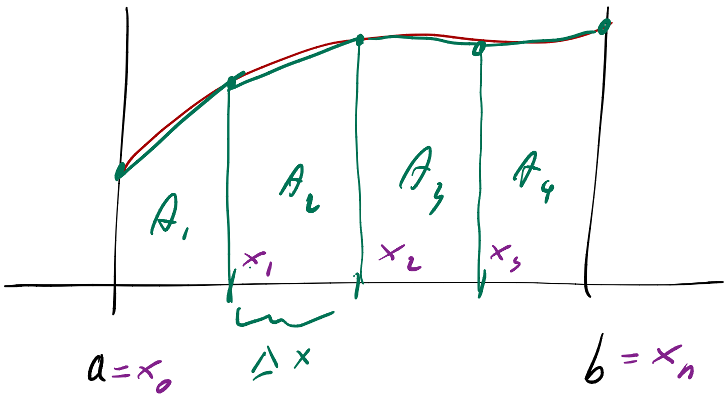
$$A = \int_a^b f(x)\,dx$$



$$\int_a^b f(x)\,dx = A_1 - A_2$$

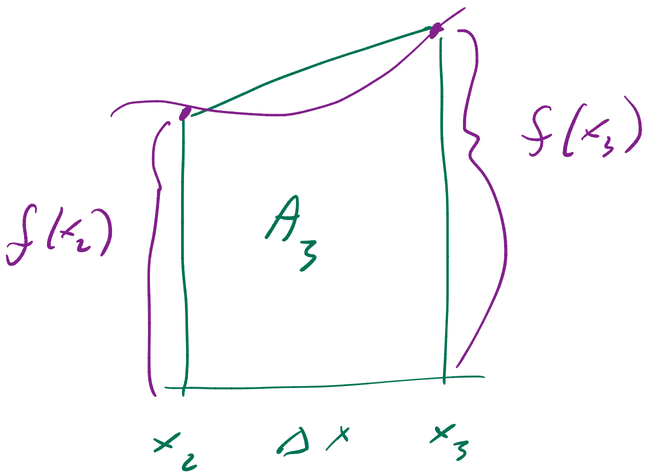
$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

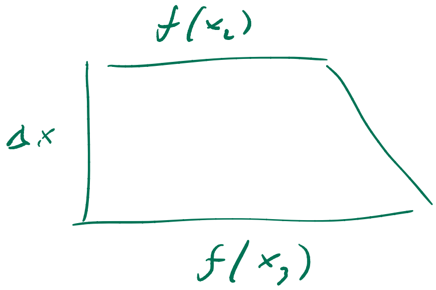
$$\int f(x)\,dx = \underbrace{\int_0^x f(x)\,dx}_{F(x)} + C$$



$$\int_a^b f(x)\,dx \approx A_1 + A_2 + A_3 + A_4$$

n trapezi $\Rightarrow$ $\Delta x = \dfrac{b-a}{n}$





$$A_3 = \Delta x \cdot \frac{f(x_2) + f(x_3)}{2}$$

$$A_1 + A_2 + A_3 + \dots + A_n =$$

$$= \Delta x \, \frac{f(x_0) + f(x_1)}{2} + \Delta x \, \frac{f(x_1) + f(x_2)}{2} + \dots$$

$$= \Delta x \left(\frac{f(x_0)}{2} + f(x_1) + f(x_2) + \dots + f(x_{n-1}) + \frac{f(x_n)}{2} \right)$$

$$x_i = a + i \cdot \Delta x$$

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p. 257 nota.

$$(A_1 + \dots + A_n) - \int_a^b f(x)\,dx \approx \Delta x^2 \, \frac{f'(b) - f'(a)}{12}$$

$$\int_0^1 e^{-x^2}\,dx = \circledast \qquad n=5 \quad \text{trapezi}$$

$$\Delta x = \frac{1-0}{5} = 0.2$$

$$\circledast \approx 0.2\left(\underbrace{\underbrace{\frac{e^{-0^2}}{2}}_{0.5} + \underbrace{e^{-0.2^2}}_{0.96} + \underbrace{e^{-0.4^2}}_{0.85} + \underbrace{e^{-0.6^2}}_{0.70} + \underbrace{e^{-0.8^2}}_{0.53} + \underbrace{\frac{e^{-1^2}}{2}}_{0.18}}_{3.72}\right)$$

$$= 0.74$$

Vediamo esercizi p. 258 – 262

$$f'' = 2f' - f + 2$$

(A) Considero l'eq. omogenea associata

$$f'' = 2f' - f$$

$$f = e^{\alpha x} \qquad f' = \alpha e^{\alpha x} \qquad f'' = \alpha^2 e^{\alpha x}$$

$$\alpha^2 \cancel{e^{\alpha x}} = 2\alpha \cancel{e^{\alpha x}} - \cancel{e^{\alpha x}}$$

$$\alpha^2 = 2\alpha - 1$$

$$\alpha^2 - 2\alpha + 1 = 0 \qquad \begin{cases} \Delta = 0 \\ \alpha = 1 \end{cases}$$

Sol. generale dell'eq. omogenea

$$f(x) = C_1 e^x + C_2 x e^x$$

(B) Trovo una sol. particolare della equazione completa

$$f'' = 2f' - f + 2$$

$$f(x) = 2$$

(c) La soluzione richiesta è

$$f(x) = C_1 e^x + C_2 x e^x + 2$$

$$f'(x) = C_1 e^x + C_2 x e^x + C_2 e^x$$

$$f''(x) = C_1 e^x + C_2 x e^x + C_2 e^x + C_2 e^x$$

$$f'' = 2f' - f + 2$$

$$2f' - f + 2 = 2C_1 e^x + 2C_2 x e^x + 2C_2 e^x$$

$$- C_1 e^x - C_2 x e^x \cancel{-2} \cancel{+2}$$

$$= e^x (C_1 + 2C_2) + C_2 x e^x$$

Trova la sol. part. con

$$^{*} f(0) = 0 \qquad ^{**} f'(0) = 0$$

$$* \quad C_1 \overset{1}{\underline{e^0}} + \cancel{C_2\, 0 \cdot e^0} + 2 = 0 \qquad C_1 = -2$$

$$** \quad (C_1 + C_2)\overset{1}{e^0} + \cancel{C_2\, 0\, e^0} = 0 \qquad C_2 = 2$$

$$f(x) = -2e^{x} + 2xe^{x} + 2$$