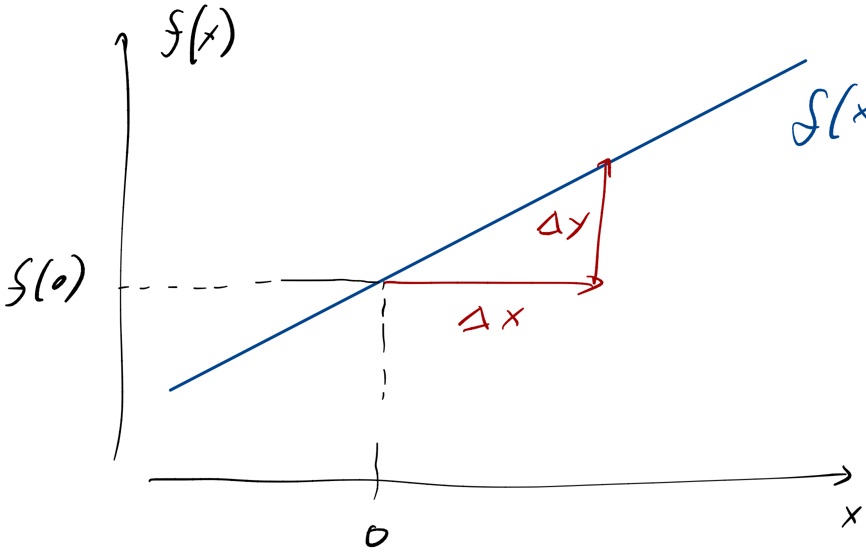
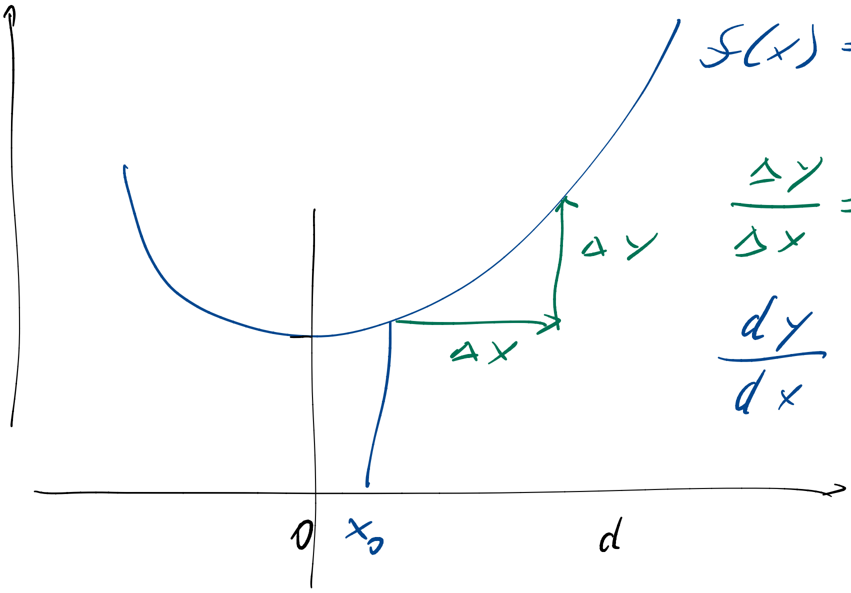
Derivative



$$f(x) = f(0) + a \cdot x$$

$$\frac{\Delta y}{\Delta x}$$



$f(x) = x^2$ $\quad f'(x) = 2x$

$$\frac{\Delta y}{\Delta x} = 2x_0 + \Delta x$$

$$\frac{dy}{dx} = 2x_0$$

$$f(x) = ax + b$$

$$x_0 \xrightarrow{+\Delta x} x_0 + \Delta x$$

$$ax_0 + b \longrightarrow ax_0 + a\Delta x + b$$

$$\Delta y = a \cdot \Delta x$$

$$\frac{\Delta y}{\Delta x} = \frac{a\Delta x}{\Delta x} = a$$

$$f(x) = x^2$$

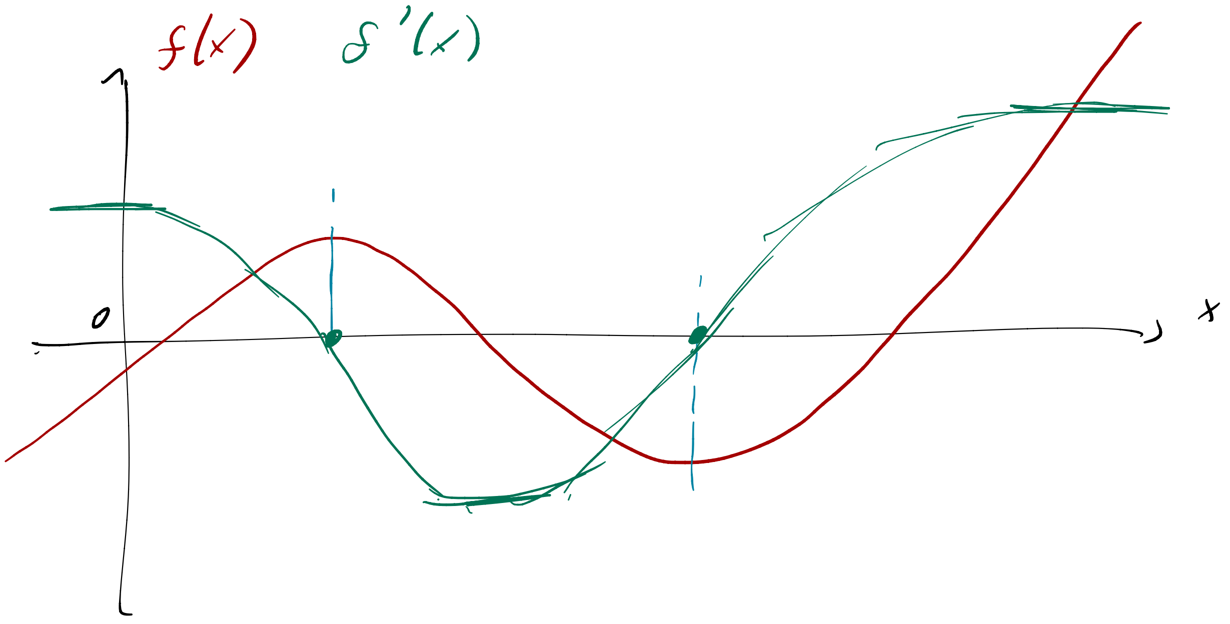
$$x_0 \xrightarrow{+\Delta x} x_0 + \Delta x$$

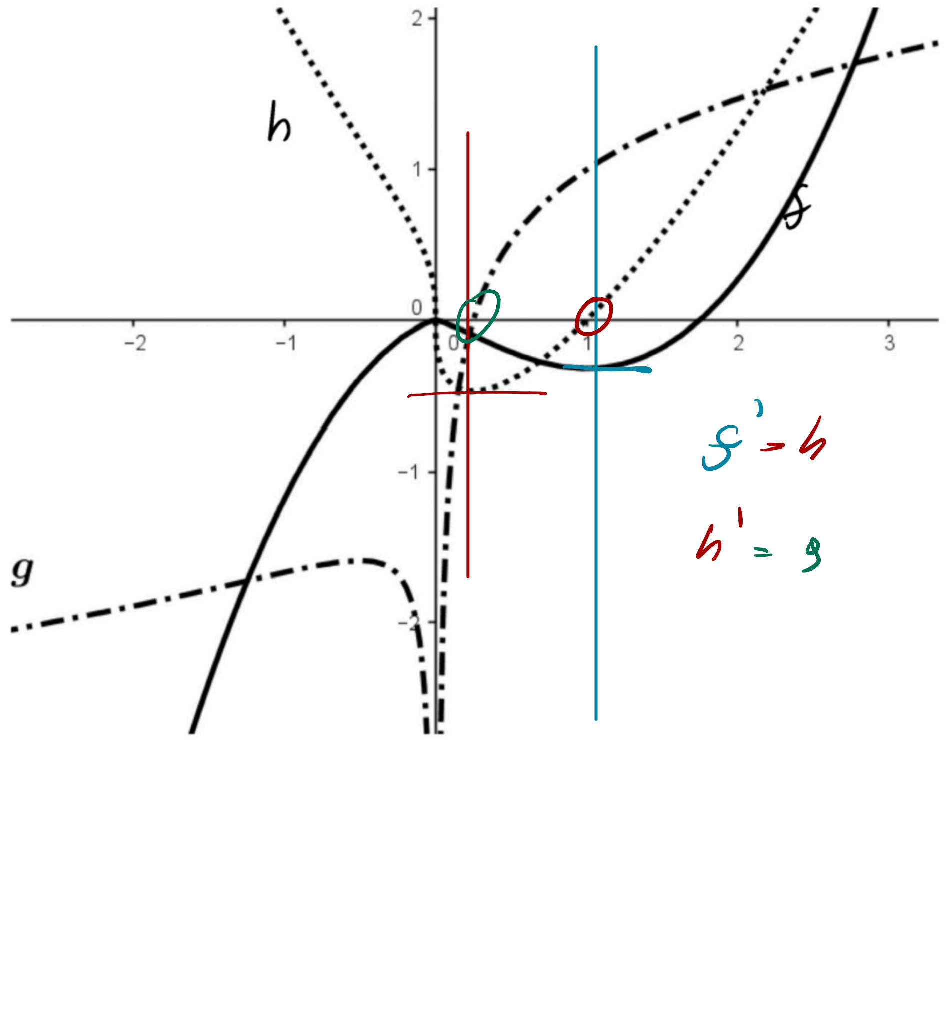
$$x_0^2 \longrightarrow (x_0 + \Delta x)^2$$

$$\Delta y = (x_0 + \Delta x)^2 - x_0^2 = 2\ x_0 \cdot \Delta x + \Delta x^2$$

$$\frac{\Delta y}{\Delta x} = \frac{2x_0\Delta x + \Delta x^2}{\Delta x} = 2x_0 + \Delta x$$

$$f(x) = x^2 \longrightarrow \frac{df}{dx} = f'(x) = 2x$$


f(x)
f '(x)
0
x


h
f
g
f ' = h
h ' = g

Calcolo delle derivate

f' ← derivata di f

$$(ax+b)' = a$$

$$(x^2)' = 2x$$

$$(x^\alpha)' = \alpha \cdot x^{\alpha-1}$$

$$(\sqrt{x})' = \left(x^{\frac{1}{2}}\right)' = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

$$(\sin x)' = \cos x$$

IN RADIANTI

$$\left(\frac{1}{x}\right)' = (x^{-1})' = -1 \cdot x^{-2} = \frac{-1}{x^2}$$

$$(\cos x)' = -\sin x$$

$$(\tan x)' = \frac{1}{(\cos x)^2} = 1 + (\tan x)^2$$

$$(e^x)' = e^x$$ ← DEFINIZIONE DI e.

$$(b^x)' = b^x \cdot \log_e b$$

$$(\log_e x)' = \frac{1}{x}$$

$$(\log_b x)' = \frac{1}{x \cdot \log_e b}$$

↓ non richiesta per il TEST

$$(\arctan x)' = \frac{1}{1+x^2}$$

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$$

$$(\arccos x)' = \frac{-1}{\sqrt{1-x^2}}$$

Regole di derivazione

$$(f(x)+g(x))' = f'(x)+g'(x)$$

$$(1+2x+\sin(x))' = (1+2x)' + (\sin(x))'$$

$$= 2 + \cos(x)$$

$$(f(x)-g(x))' = f'(x)-g'(x)$$

$$(f(x)\cdot g(x))' = f'(x)\cdot g(x) + f(x)\cdot g'(x)$$

$$(x\cdot \sin x)' = 1\cdot \sin(x) + x\cdot\cos(x)$$

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{(g(x))^2}$$

$$\left(\frac{\sin(x)}{x}\right)' = \frac{\cos(x)\cdot x - \sin(x)}{x^2}$$

$$(k\cdot f(x))' = \overset{0}{k'}\cdot f(x) + k\cdot f'(x)$$

$$= k\cdot f'(x)$$

Bs

$$(\sin(x)\cos(x))' = \overset{\cos(x)}{\sin'(x)}\cdot\cos(x) + \sin(x)\cdot\overset{-\sin(x)}{\cos'(x)}$$

$$= (\cos(x))^2 - (\sin(x))^2$$

$$\left((x^2+2)\cdot e^{7x}\right)' = (x^2+2)'\cdot e^{7x} + (x^2+2)\cdot(e^{7x})'$$

$$= \left((x^2)' + 2'\right)\cdot e^{7x} + (x^2+2)\left((e^7)^x\right)'$$

$$= (2x+0)\, e^{7x} + (x^2+2)(e^7)^x \log_e e^7$$

$$= 2x e^{7x} + (x^2+2)\, e^{7x}\cdot 7$$

$$= (7x^2+2x+14)\, e^{7x}$$

$$(f(g(x)))' = f'(g(x)) \cdot g'(x)$$

$$(\underbrace{\sin}_{f}(\underbrace{2x+1}_{g}))' = \underbrace{\cos}_{f'}(\underbrace{2x+1}_{g}) \cdot \underbrace{2}_{g'}$$

$$(f(ax+b))' = a\, f'(ax+b)$$

$$(e^{x^2})' \qquad\qquad x \overset{g}{\longmapsto} x^2 \overset{f}{\longmapsto} e^x$$

$$= e^{x^2} \cdot 2x \qquad\qquad (f(g(x)))' = f'(g(x)) \cdot g'(x)$$

$$(\underbrace{\sin(x)\cos(x)}_{\frac{1}{2}\sin(2x)})' = \left(\frac{1}{2}\sin(2x)\right)'$$

$$= \frac{1}{2}(\sin(2x))'$$

$$\cos(2x) = \frac{1}{2} \cdot 2\cos(2x)$$

$=$ ☺

$$= (\cos(x))^2 - (\sin(x))^2$$

$$f(x) = \big(x(x-1)\big)(x+1)$$

$$f'(x) = (x^3 - x)' = 3x^2 - 1$$

$$(x^x)' = \left(e^{x \cdot \log_e x}\right)' = *$$

$$e^{x \cdot \log_e(x)}$$

$$x \xmapsto{g} x \cdot \log_e x \xmapsto{f} e^{\square}$$

$$* = e^{x \cdot \log_e x} \cdot \underbrace{(x \cdot \log_e x)'}_{1 \cdot \log_e x + \cancel{x}\frac{1}{\cancel{x}}} =$$

$$= x^x (1 + \log_e x)$$

$$f(x) = \log_e(x^4)$$

$$f'(x) = (4 \cdot \log_e x)' = \frac{4}{x}$$

$$x \xmapsto{g} x^4 \xmapsto{f} \log_e(x^4)$$

$$(*)' = \frac{1}{x^{\cancel{4}}} \cdot 4\cancel{x^3} = \frac{4}{x}$$