ESERCIZIO D 23 · VI · 2025

Trova la soluzione particolare dell'equazione differenziale

$$f'' = -2f' - 5f$$

tale che $f(0) = 1$ e $f'(0) = 0$.

$$f(x) = e^{\lambda x} \qquad f'(x) = \lambda e^{\lambda x} \qquad f'' = \lambda^2 e^{\lambda x}$$

$$\lambda^2 \cancel{e^{\lambda x}} = -2\lambda \cancel{e^{\lambda x}} - 5 \cancel{e^{\lambda x}}$$

$$\lambda^2 + 2\lambda + 5 = 0$$

$$\lambda = \frac{-2 \pm \sqrt{\Delta}}{2}$$

$$\Delta = 2^2 - 4 \cdot 5 = -16$$

$$f(x) = e^{-x}\left(C_1 \cos(\omega x) + C_2 \sin(\omega x)\right)$$

$$\frac{\sqrt{-\Delta}}{2} = 2$$

$$f(x) = e^{-x}\left(C_1 \cos(2x) + C_2 \sin(2x)\right)$$

SOLUZIONE GENERALE

$$f'(x) = e^{-x}\left(-2C_1 \sin(2x) + 2C_2 \cos(2x)\right) + - e^{-x}\left(C_1 \cos(2x) + C_2 \sin(2x)\right)$$

$$\begin{cases} f(0) = 1 \\ f'(0) = 0 \end{cases} \qquad \begin{array}{l} C_1 = 1 \\ 2C_2 - C_1 = 0 \rightarrow C_2 = \frac{1}{2} \end{array}$$

$$f(x) = e^{-x}\left(\cos(2x) + \frac{1}{2}\sin(2x)\right)$$

$$f'(x) = e^{-x}\left(-2\sin(2x) + \cos(2x)\right)$$

$$- e^{-x}\left(\cos(2x) + \frac{1}{2}\sin(2x)\right)$$

$$= -\frac{5}{2} e^{-x}\sin(2x)$$

$$f''(x) = -5e^{-x}\cos(2x) + \frac{5}{2}e^{-x}\sin(2x)$$

$$-2f'(x) - 5f(x) =$$

$$= 5e^{-x}\sin(2x) - 5e^{-x}\cos(2x)$$

$$-\frac{5}{2}e^{-x}\sin(2x)$$

$$f''(x) = -2f'(x) - 5f(x)$$

Esercizio D 19 · V · 2025

Trova la soluzione particolare dell'equazione differenziale $f''(x) = f(x)$ tale che $f(5) = 1$ e $f'(5) = 0$. Rappresenta quindi questa funzione tramite un diagramma semi-logaritmico, per $0 \leqslant x \leqslant 10$.

$$f''(x) = f(x) \qquad f(x) = e^{\lambda x}$$

$$\lambda^2 = 1$$

$$\lambda = \begin{cases} +1 \\ -1 \end{cases}$$

$$f(x) = C_1 e^{x} + C_2 e^{-x}$$

$$\begin{cases} f(5) = 1 \\ f'(5) = 0 \end{cases} \qquad \begin{cases} C_1 e^{5} + C_2 e^{-5} = 1 \\ C_1 e^{5} - C_2 e^{-5} = 0 \end{cases}$$

$$C_2 = C_1 \cdot e^{10}$$

$$C_1 e^{5} + C_1 e^{5} = 1 \implies C_1 = \frac{1}{2} e^{-5}$$

$$C_2 = \frac{1}{2} e^{5}$$

$$f(x) = \frac{1}{2} e^{x-5} + \frac{1}{2} e^{5-x}$$

Esercizio D $7 \cdot 11 \cdot 2025$

Scrivi un'equazione differenziale lineare del secondo ordine a coefficienti costanti che ammetta la soluzione particolare $f_1(x) = e^{2x} - e^{-x} + 2$. Determina quindi la soluzione particolare $f_2(x)$ di questa equazione che soddisfa $f_2(0) = f_2'(0) = 0$.

$$f''(x) = a f'(x) + b f(x) + c$$

$$f(x) = e^{2x} - e^{-x} + 2$$

$$f'(x) = 2e^{2x} + e^{-x}$$

$$f''(x) = 4e^{2x} - e^{-x}$$

$$4e^{2x} - e^{-x} = a\left(2e^{2x} + e^{-x}\right) + b\left(e^{2x} - e^{-x} + 2\right) + c$$

$$4e^{2x} - e^{-x} =$$

$$= \underbrace{(2a+b)}_{4} e^{2x} + \underbrace{(a-b)}_{-1} e^{-x} \underbrace{+ 2b + c}_{0}$$

$$\begin{cases} 2a + b = 4 \\ a - b = -1 \\ 2b + c = 0 \end{cases} \qquad \begin{aligned} a &= 1 \\ b &= 2 \\ c &= -4 \end{aligned}$$

$$\boxed{f'' = f' + 2f - 4}$$

$$f(x) = C_1 e^{2x} + C_2 e^{-x} + 2$$

$$\begin{cases} f(0) = 0 \\ f'(0) = 0 \end{cases} \qquad \begin{cases} C_1 + C_2 + 2 = 0 \\ 2C_1 - C_2 = 0 \end{cases}$$

$$\Downarrow$$

$$C_1 = -\frac{2}{3} \qquad C_2 = -\frac{4}{3}$$

$$f(x) = -\frac{2}{3} e^{2x} - \frac{4}{3} e^{-x} + 2$$

Eq. a variabili separabili

$$f'(x) = g(x) \cdot h(f(x))$$

$$\frac{df}{dx} = g(x)\, h(f)$$

$$\int \frac{df}{h(f)} = \int g(x)\, dx$$

$$f'(x) = x\, f(x)$$

$$\frac{df}{dx} = x \cdot f$$

$$\int \frac{df}{f} = \int x\, dx$$

$$\log f = \frac{1}{2}x^2 + C$$

$$f(x) = e^{\frac{1}{2}x^2 + C}$$

$$f'(x) = e^{\frac{1}{2}x^2 + C} \cdot x$$

$$f' = f \cdot x$$

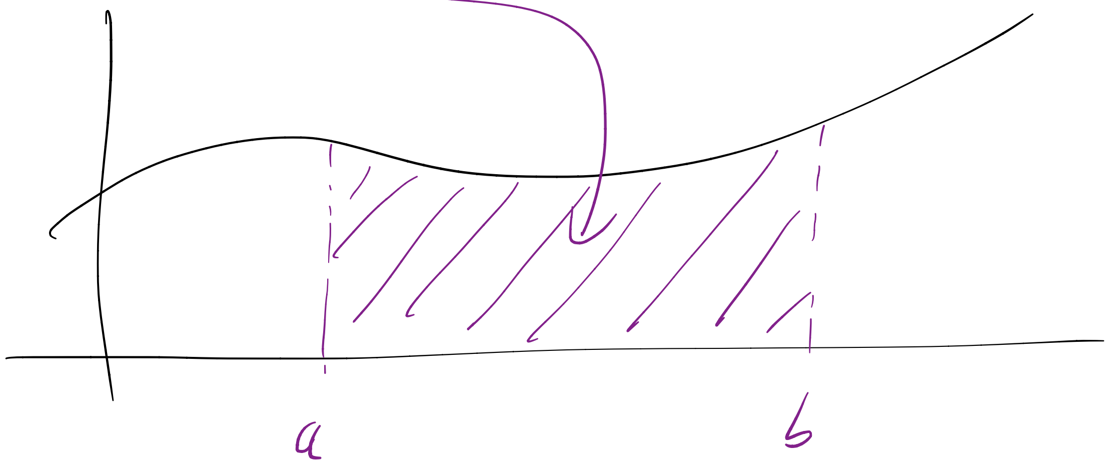
$$f(x) = e^{\frac{1}{2}x^2 + C}$$

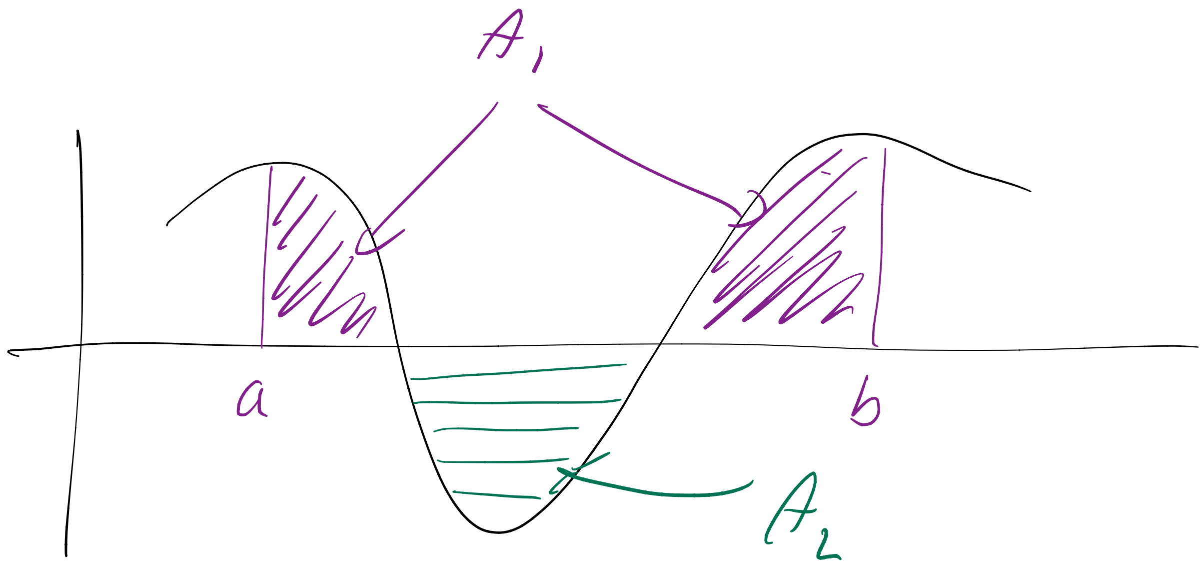
$$= e^{C} \cdot e^{\frac{1}{2}x^2}$$

$$= K \cdot e^{\frac{1}{2}x^2}$$

Integrali definiti

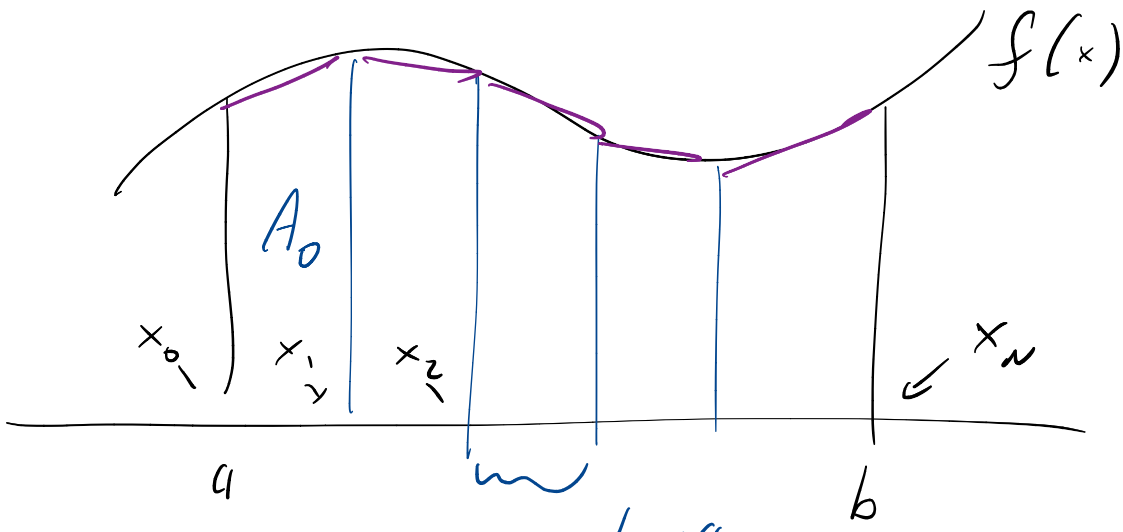
$$\underbrace{\int_a^b f(x)\,dx}_{} = F(b) - F(a)$$





$$\int_a^b f(x) = A_1 - A_2$$

Metodo dei trapezi



$$\Delta x = \frac{b-a}{N}$$

$$A_0 = \Delta x \cdot \frac{f(x_0) + f(x_1)}{2}$$

$$A_1 = \Delta x \cdot \frac{f(x_1) + f(x_2)}{2}$$

$$\int_a^b f(x)\,dx$$

$\vdots$

$$A = \Delta x \left(\frac{f(x_0)}{2} + f(x_1) + f(x_2) + \dots + \frac{f(x_N)}{2} \right)$$

$$x_i = a + i \cdot \Delta x$$