Funzioni lineari

$$f(x) = ax + b$$

Es. Determinare f lineare t.c.

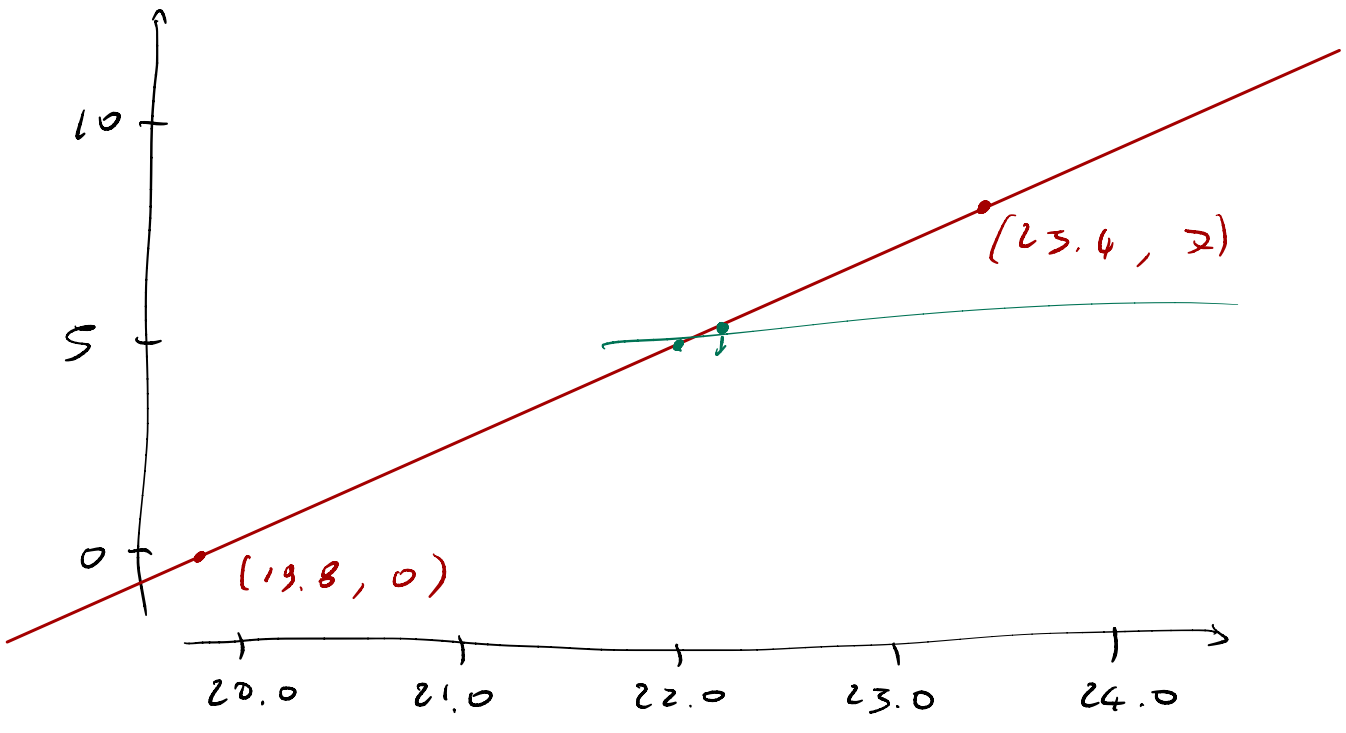
$$\begin{cases} f(2) = 5 \\ f(10) = 7 \end{cases} \qquad f(x) = ax + b$$

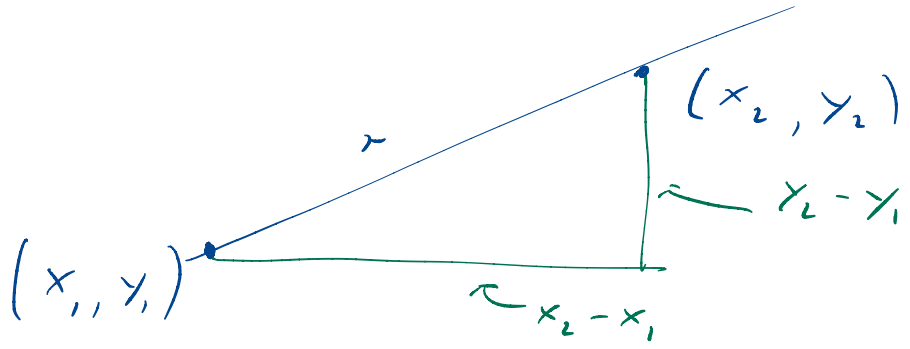
$$\begin{cases} a\,2 + b = 5 \\ a\,10 + b = 7 \end{cases} \qquad \frac{1}{4} \cdot 2 + b = 5$$

$$a \cdot 8 = 2 \implies a = \frac{1}{4}$$

$$b = \frac{9}{2}$$

$$f(x) = \frac{1}{4}x + \frac{9}{2}$$

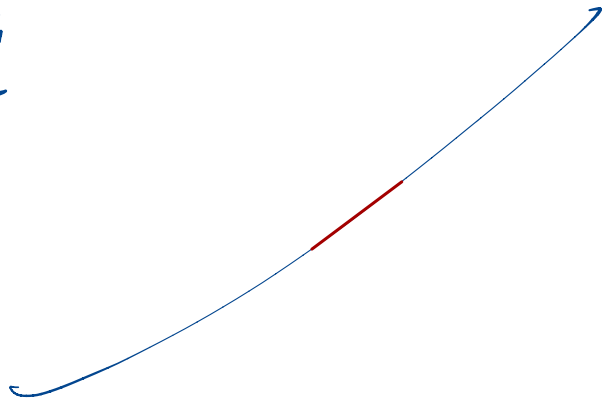




F. di interpolazione lineare

$$y = y_1 + \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

	x	$f(x)$	
x_1	0.50	35.21	y_1
x_2	0.55	34.29	y_2
	0.60	33.32	
	0.65	32.30	
	0.70	31.23	



$$f(0.5200) \approx 38.29 + \frac{41.77 - 38.29}{0.55 - 0.50}(0.52 - 0.50)$$

$$x = x_1 + \frac{x_2 - x_1}{y_2 - y_1}(y - y_1)$$

Funzione esponenziale

$$a^n = \underbrace{a \cdot a \cdot \ldots \cdot a}_{n \text{ volte}}$$

$$a^{-n} = \frac{1}{a^n} \qquad a^{\frac{1}{n}} = \sqrt[n]{a}$$

a^x ha senso per $a > 0$ e x qualunque

$$a^0 = 1 \qquad a^x \cdot a^y = a^{x+y} \qquad a^x \cdot b^x = (ab)^x \qquad (a^x)^y = a^{x \cdot y}$$

$$a^1 = a \qquad \frac{a^x}{a^y} = a^{x-y} \qquad \frac{a^x}{b^x} = \left(\frac{a}{b}\right)^x$$

$$1^x = 1$$

Sapendo che $8^x = 2.7 \cdot 10^4$, quanto vale 2^x?

$$2.7 \cdot 10^4 = 8^x = \left(2^3\right)^x = 2^{3x} = 2^{x \cdot 3}$$

$$\left(2^x\right)^3$$

$$2^x = \sqrt[3]{2.7 \cdot 10^4} = \sqrt[3]{27 \cdot 10^3} = 3 \cdot 10 = 30$$

35 – Sapendo che $10^{3x} = 64$ quanto vale 100^x?

$64 = 10^{3x} = (10^x)^3 \Leftarrow 10^x = \sqrt[3]{64} = 4$

$100^x = (10^2)^x = (10^x)^2 = 4^2 = 16$

37 – Metti in ordine crescente $4^{1.5}$, $10^{0.5}$, 5.

$4^{1.5} = 4^1 \cdot 4^{0.5} = 4 \cdot \sqrt{4} = 8$

$10^{0.5} = \sqrt{10} = 3.\ldots$

5

$10^{0.5} < 5 < 4^{1.5}$

38 – Scrivi $7 \cdot 0.3^{-2}$ approssimato a tre cifre significative.

$7 \cdot 0.3^{-2} = 7 \cdot \frac{1}{0.3^2} = 7 \cdot \frac{1}{\frac{3^2}{10^2}} =$

$= \frac{7 \cdot 10^2}{3^2} = \frac{700}{9} = 700 \cdot 0.111\ldots$

$\frac{1}{9} = 0.111\ldots$ $= 77.777\ldots \approx 77.8$

42 – Risolvi la disequazione $2^x < 3^x$.

$0 < x$

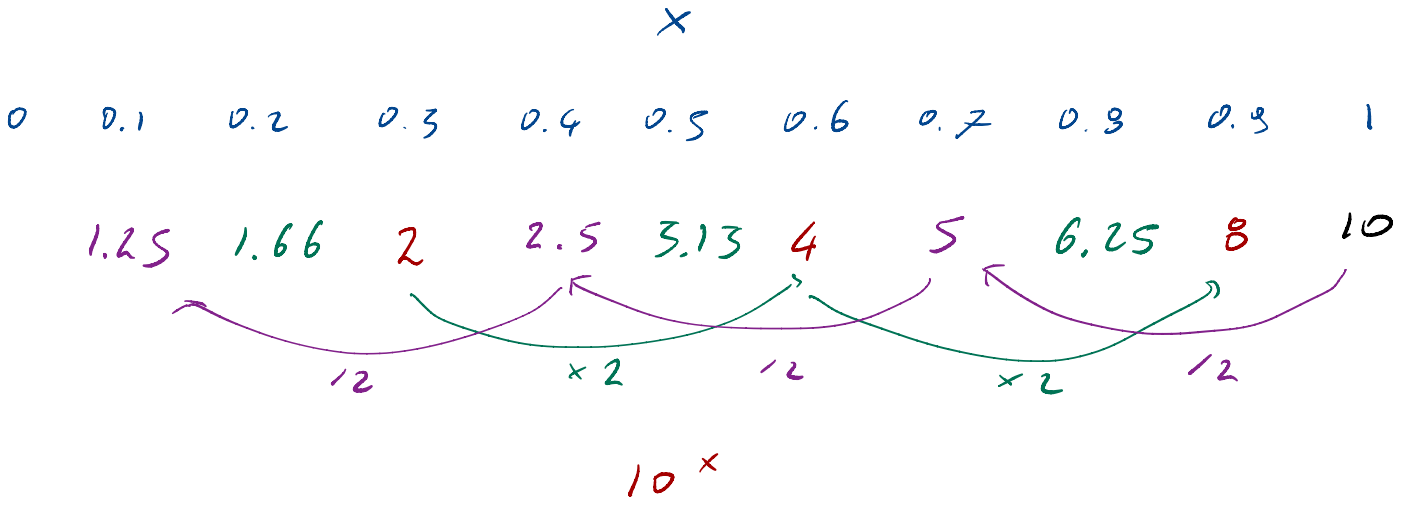
Potenze di 10

$10^{4.6} \approx 40\,000$

$10^{0.3} \approx 2$

$10^{4} \cdot 10^{0.6} = 10\,000 \cdot \left(10^{0.3}\right)^2$

$\approx 10\,000 \cdot 2^2 = 40\,000$



10^x

$10^{0.7} = 10^{1-0.3} = \frac{10^1}{10^{0.3}} \approx \frac{10}{2} = 5$

Funzione logaritmo

$$\log_b x = y \iff b^y = x$$

Es $\log_{10} 100 = 2$ $\quad (10^2 = 100)$

$\log_{10} 2 \approx 0.3$ $\quad (10^{0.3} \approx 2)$

$\log_b 1 = 0$ $\qquad \log_b (x \cdot y) = \log_b x + \log_b y$

$\log_b b = 1$ $\qquad \log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y$

$\log_b (x^y) = y \cdot \log_b x$ $\qquad \log_b x = \dfrac{\log_\square x}{\log_\square b}$

49 – Calcola $\log_2\left(\frac{8^{3.7}}{4}\right)$.

$$\log_2\left(\frac{8^{3.7}}{4}\right) = \log_2\left(8^{3.7}\right) - \log_2(4)$$

$$= 3.7 \cdot \underbrace{\log_2(8)}_{3} - \underbrace{\log_2(4)}_{2}$$

$$(2^3 = 8) \qquad (2^2 = 4)$$

$$= 3.7 \cdot 3 - 2 = 9.1$$

55 – Se $\log_{10}(x^2) = 4.2$, quanto vale $\log_{10}(10\,x)$?

$$4.2 = \log_{10}(x^2) = 2 \cdot \log_{10}(x) \leftarrow 2.1$$

$$\log_{10}(10 \cdot x) = \underbrace{\log_{10}(10)}_{1} + \log_{10}(x) = 3.1$$