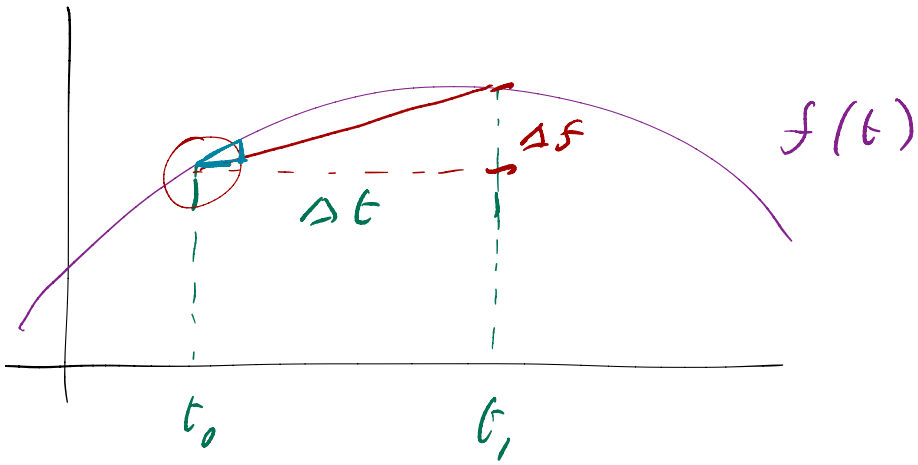
Derivate

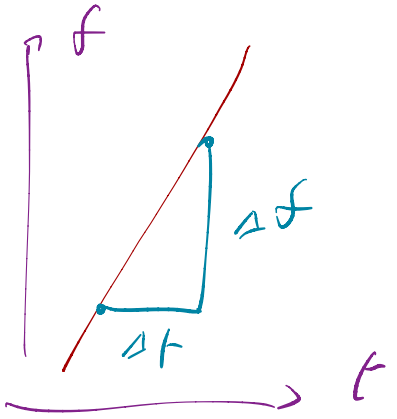


$$\frac{\Delta f}{\Delta t} = \frac{f(t_1) - f(t_0)}{t_1 - t_0}$$

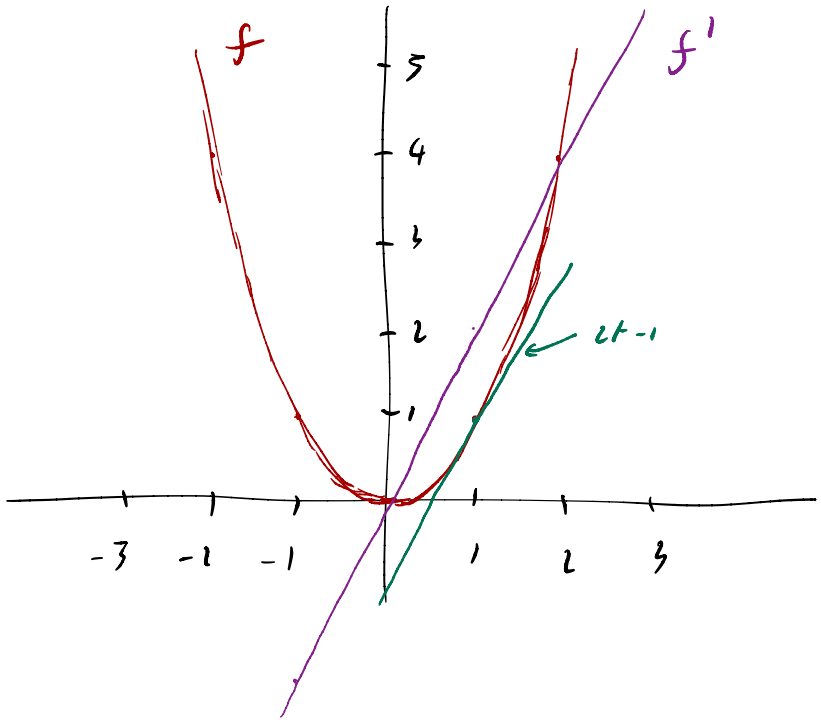
$$f'(t_0) = \frac{f(t_1) - f(t_0)}{t_1 - t_0} = \frac{df}{dt}$$

infinitesimamente lontano da t_0

Es. $f(t) = qt + 2$ $\qquad f'(t) = q$



Es $f(t) = t^2$ $\quad f'(t) = 2t$



Derivate di alcune funzioni elementari

$(\text{costante})' = 0$

$(x^n)' = n \cdot x^{n-1}$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(e^x)' = e^x$

$x^{\frac{1}{n}}$

$(\sqrt[n]{x})' = \frac{1}{n} \cdot \sqrt[n]{x^{n-1}}$

$(\log_e x)' = \frac{1}{x}$

Regole di calcolo

$$(\text{cost.}\ f(x))' = \text{cost.}\ f'(x)$$

Es $f(x) = 2\sin(x)$ $\quad f'(x) = 2\dfrac{\cos(x)}{(\sin x)'}$

$$(f(x)+g(x))' = f'(x) + g'(x)$$

Es $f(x) = x^2 + x^3$ $\quad f'(x) = 2x + 3x^2$

$$(f(x)\cdot g(x))' = f'(x)\cdot g(x) + f(x)\cdot g'(x)$$

Es. $f(x) = x\cdot\sin(x)$

$$f'(x) = (x)'\cdot\sin(x) + x\cdot(\sin(x))'$$

$$= \sin(x) + x\cdot\cos(x)$$

$$(f(g(x)))' = f'(g(x))\cdot g'(x)$$

6) $h(x) = \sin(x^2)$

$$x \xrightarrow[g]{\square^2} x^2 \xrightarrow[f]{\sin(\cdot)} h(x)$$

$$h(x) = f(g(x))$$

$$h' = f'(g(x)) \cdot g'(x)$$

$$= \cos(x^2) \cdot 2x = \underline{2x\cos(x^2)}$$

~~=~~

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{(g(x))^2}$$

$$\frac{f(x)}{g(x)} = f(x)\cdot(g(x))^{-1}$$

$$= f'(x)\cdot(g(x))^{-1} + f(x)\cdot\left((g(x))^{-1}\right)'$$

$\left((g(x))^{-1}\right)'$: $\frac{-1}{(g(x))^2}\cdot g'(x)$

$$\ldots = f'(x)\cdot(g(x))^{-1} - f(x)\cdot\frac{g'(x)}{(g(x))^2}$$

$$= \frac{f'(x)\cdot g(x) - f(x)\,g'(x)}{(g(x))^2}$$

$$\left((g(x))^{-1}\right)'$$

$$x \xrightarrow{\ g\ } g(x) \xrightarrow{\ \square^{-1}\ } (g(x))^{-1}$$

g : f

$$(x^{-1})' = \frac{-1}{x^2}$$

$$(f(g(x)))' = f'(g(x)) \cdot g'(x)$$

$$= \frac{-1}{(g(x))^2} \cdot g'(x)$$

Ex $\left(\sin(2x)\right)'$

$$x \xrightarrow{\ 2\cdot\square\ } 2x \xrightarrow{\ \sin\ } \sin(2x)$$

g : f

$$\left(\sin(2x)\right)' = \left(f(g(x))\right)'$$

$$= f'(g(x)) \cdot \underbrace{g'(x)}_{2} = 2\cos(2x)$$

$\cos$

costante

$$\left(f(k\cdot x)\right) = k\cdot f'(kx)$$

Es $\left(10^x\right)'$ $\qquad 10 = e^{\log_e(10)}$

$$\left(\left(e^{\log_e(10)}\right)^x\right)' = \left(e^{x\cdot\log_e(10)}\right)' =$$

$$= \log_e(10)\cdot \underbrace{e^{x\cdot\log_e(10)}}_{10^x} = \log_e(10)\cdot 10^x$$

Es $\left(\log_{10} x\right)' = \left(\frac{\log_e x}{\log_e 10}\right)' =$

$$= \left(\underbrace{\frac{1}{\log_e 10}}_{\text{cost.}}\cdot \log_e x\right)' = \frac{1}{\log_e 10}\cdot\left(\log_e x\right)'$$

$$= \frac{1}{x\cdot\log_e 10}$$

Es $\left(x^{x^2} \right)'$ $\qquad x = e^{\log_e x}$

$\| $

$\left(e^{\log_e(x) \cdot x^2} \right)' = (*)$

$x \xmapsto{\log_e(x)\cdot x^2} \cdots \xmapsto{e^{\square}} e^{\cdots}$

$g \qquad\qquad f$

$(*) = \underbrace{e^{\log_e(x)\cdot x^2}}_{x^{x^2}} \cdot \left(\log_e(x) \cdot x^2 \right)'$

$= x^{x^2} \cdot \left(\left(\log_e(x)\right)' \cdot x^2 + \log_e(x) \cdot \left(x^2\right)' \right)$

$\left(\log_e(x)\right)' = \frac{1}{x} \qquad \left(x^2\right)' = 2x$

$= x^{x^2} \left(x + 2x \log_e(x) \right) =$

$= x^{1+x^2} \left(1 + 2\log_e(x) \right)$

Notazioni

$$f(x) \qquad f'(x) = \left(f(x)\right)' = \frac{df}{dx}$$

$$P = \frac{nRT}{V}$$

$$\frac{\partial P}{\partial V} = -\frac{nRT}{V^2} \qquad \frac{\partial P}{\partial T} = \frac{nR}{V}$$

$$\left[\frac{\partial P}{\partial V}\right] = \frac{[P]}{[V]} = \frac{Pa}{m^3}$$

($[P] = Pa$, $[V] = m^3$)

$$[a] = \left[\frac{dv}{dt}\right] = \frac{[v]}{[t]} = \frac{m/s}{s} = \frac{m}{s^2}$$

Derivata seconda

$$\left(f'(x)\right)' = f''(x) = \frac{d^2 f}{dx^2}$$

Esercizio 133 (soluzione a p. 329)

Per ciascuna di queste coppie di grafici decidi se f potrebbe essere la derivata o g quella di f, oppure non si dà nessuno di questi due casi.

