Trova un polinomio di terzo grado $f(x)$ tale che

$$x^2 + x f'(x) = 3 f(x)$$

$$f(x) = ax^3 + bx^2 + cx + d$$

$$f'(x) = 3ax^2 + 2bx + c$$

$$x^2 + x(3ax^2 + 2bx + c) = 3(ax^3 + bx^2 + cx + d)$$

$$3ax^3 + (2b+1)x^2 + cx = 3ax^3 + 3bx^2 + 3cx + 3d$$

$$\begin{cases} 2b+1 = 3b \rightarrow b = 1 \\ c = 3c \rightarrow c = 0 \\ 0 = 3d \rightarrow d = 0 \end{cases}$$

Vanno bene tutti i polinomi nella forma

$$f(x) = ax^3 + x^2$$

↑

soluzione generale di

$$x^2 + x f'(x) = 3 f(x)$$

# Integrali indefiniti

$$f'(x) = x^4$$

↑ una funzione della $x$.

$$f(x) = \frac{x^5}{5} + C$$

$$\int x^4 \, dx = \frac{x^5}{5} + C$$

$$x \cdot \tan\left(e^{\sqrt{x}} + \log\left(\frac{1}{x}\right)\right)$$

$$\int e^{-x^2} \, dx$$

# Integrali di f. elementari

$$\int x^\alpha \, dx = \frac{1}{\alpha+1} x^{\alpha+1} + C \qquad \text{per} \quad \alpha \neq -1$$

$$\int \frac{1}{x} \, dx = \log_e x + C$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \cos x \, dx = \sin x + C \qquad \int x^0 \, dx$$

$$\int e^x \, dx = e^x + C \qquad \int 1 \, dx = x + C$$

# Regole di calcolo

$$\int f(x) \, dx = F(x) + C$$

$$\int g(x) \, dx = G(x) + C$$

$$\int k \cdot f(x) \, dx = k \cdot F(x) + C$$

$$= k \cdot \int f(x) \, dx$$

$$\int f(x) + g(x)\ dx = F(x) + G(x) + C$$

$$= \int f(x)\,dx + \int g(x)\,dx$$

Ex. $\int x - \sin(x)\ dx =$

$= \int x\,dx + \int -\sin(x)\ dx$ ($-1 \cdot \sin(x)$)

$= \frac{1}{2}x^2 + C_1 - \int \sin(x)\,dx$

$= \frac{1}{2}x^2 + C_1 + \cos(x) - C_2$

$= \frac{1}{2}x^2 + \cos(x) + (C_1 - C_2)$ ($C$)

$= \frac{1}{2}x^2 + \cos(x) + C$

# Regola di sostituzione

$$\int f(g(x)) \cdot g'(x) = F(g(x)) + C$$

Es

$$\int x \cdot \sin(x^2)\, dx =$$

$$= \int \frac{1}{2} \cdot 2x \cdot \sin(x^2)\, dx$$

$$= \frac{1}{2} \int 2x \cdot \sin(x^2)\, dx$$

$$= -\frac{1}{2} \cos(x^2) + C$$

$$\left(-\frac{1}{2} \cos(x^2) + C\right)' =$$

$$= +\frac{1}{2} \cdot +\sin(x^2) \cdot 2x + 0 = \sin(x^2) \cdot x$$

$$\int \cos(4x-7)\,dx =$$

$$= \int \frac{1}{4} \cdot 4 \cdot \cos(4x-7)\,dx$$

$$= \frac{1}{4} \int 4 \cdot \cos(4x-7)\,dx$$

$$= \frac{1}{4} \sin(4x-7) + C$$

$$\int f(ax+b)\,dx = \frac{1}{a} F(ax+b) + C$$

$$10 = e^{\log_e 10}$$

$$\int 10^x\,dx =$$

$$= \int \left(e^{\log_e 10}\right)^x dx = \int e^{(\log_e 10)\cdot x}\,dx$$

$$= \frac{1}{\log_e 10} \cdot e^{(\log_e 10)\cdot x} + C$$

$$= \frac{1}{\log_e 10}\, 10^x + C$$

$$\int \frac{x-1}{x}\,dx = \int \frac{x}{x} - \frac{1}{x}\,dx \qquad (x \neq 0)$$

$$= \int 1 - \frac{1}{x}\,dx = \int 1\,dx - \int \frac{1}{x}\,dx$$

$$= x - \log_e x + C$$

$$\int \frac{x}{x+1}\,dx = \int \frac{x+1-1}{x+1}\,dx \qquad (x \neq -1)$$

$$= \int 1 - \frac{1}{x+1}\,dx$$

$$= x - \log_e(x+1) + C$$

$$\int \frac{1}{x^2-1}\,dx$$

$$\frac{\frac{-1}{2}}{x+1} + \frac{\frac{1}{2}}{x-1}$$

$$= \int \frac{1}{(x+1)(x-1)}\,dx$$

$$= \int \frac{-\frac{1}{2}}{x+1}\,dx + \int \frac{\frac{1}{2}}{x-1}\,dx$$

$$= -\frac{1}{2}\log_e(x+1) + \frac{1}{2}\log_e(x-1) + C$$

$$= \frac{1}{2}\log_e\left(\frac{x-1}{x+1}\right) + C$$

# Equazioni lineari del 1 ord. a coeff. costanti

$$f'(x) = a f(x) + b$$

Trucco:

(1) Risolvo l'equazione omogenea associata

$$f'(x) = a f(x)$$

(2) Cerco una soluzione del tipo

$$f(x) = e^{\alpha \cdot x}$$

$$\alpha e^{\alpha x} = a \cdot e^{\alpha x} \Rightarrow \alpha = a$$

(3) La sol. generale della omogenea è

$$f(x) = C \cdot e^{ax}$$

(4) Cerco una sol. particolare della equazione data

$f'(x) = 0 \qquad 0 = a f(x) + b$

$f(x) = -\frac{b}{a}$ ⇐ sol. part. dell'eq. data

$$f(x) = C e^{ax} - \frac{b}{a}$$

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c) $f'(x) = 4 f(x) - 2$

$$f(x) = C e^{4x} + \frac{1}{2}$$

$4f - 2 = 0$

$f'(x) = 4 C e^{4x} + 0$

$4 f(x) - 2 = 4 C e^{4x} + 4 \cdot \frac{1}{2} - 2$