Teoria degli errori

errore assoluto

massa = 7.21 kg ± 0.02 kg

"probabilmente (7.21 − 0.02) kg ≤ massa ≤ (7.21 + 0.02) kg"

intervallo di incertezza

Regola

err. assoluto	int. di inc.
$x = x_0 \pm \delta x$	$\Rightarrow \quad x_0 - \delta x \le x \le x_0 + \delta x$
$x = \frac{a+b}{2} \pm \frac{b-a}{2}$	$\Leftarrow \quad a \le x \le b$

semiampiezza dell'intervallo

Es. $10.6\ g \le x \le 11.2\ g$

$$x = \frac{10.6 + 11.2}{2}\ g \pm \frac{11.2 - 10.6}{2}\ g$$

$$= (10.9 \pm 0.3)\ g$$

Propagazione degli errori

Regola della regole

L'intervallo di incertezza del risultato ha per estremi il maggiore e il minimo dei valori ottenibili al variare dei dati nei loro int. di incertezza.

Es. $x = 4.0 \pm 0.4$ $\qquad y = x + \sqrt{x}$

$$3.6 \le x \le 4.4$$

$$5.497 \le y \le 6.498$$

$$y = \frac{5.497 + 6.498}{2} \pm \frac{6.498 - 5.497}{2}$$

$$= 5.998 \pm 0.500 = 6.0 \pm 0.5$$

$$4.0 + \sqrt{4.0}$$

$297.18 \pm 0.14 \;|\; 66 = 297.18 \pm 0.15$

$x = 8.2 \pm 0.2$ $\qquad$ $y = 1.5 \pm 0.3$

$$x - y = 6.7 \pm ? \qquad \frac{2.2 - 6.2}{2} \qquad 0.5$$

y \ x	8.0	8.4
1.2	6.8	7.2
1.8	6.2	6.6

$$(x_0 \pm \delta x) + (y_0 \pm \delta y) = (x_0 + y_0) \pm (\delta x + \delta y)$$

$$(x_0 \pm \delta x) - (y_0 \pm \delta y) = (x_0 - y_0) \pm (\delta x + \delta y)$$

Errore relativo

$x_0 \pm \delta x \quad \rightarrow \quad x_0 \pm \frac{\delta x}{x_0} \cdot 100\,\%$

$10.0 \pm 0.2 \quad \rightarrow \quad 10.0 \pm 2\,\%$

$x_0 \pm \frac{x_0 \cdot \alpha}{100} \quad \leftarrow \quad x_0 \pm \alpha\,\%$

$7.0 \pm 0.7 \quad \leftarrow \quad 7.0 \pm 10\,\%$

$(x_0 \pm \alpha\,\%) \cdot (y_0 \pm \beta\,\%) = (x_0 \cdot y_0) \pm (\alpha + \beta)\,\%$

$$\frac{x_0 \pm \alpha\,\%}{y_0 \pm \beta\,\%} = \frac{x_0}{y_0} \pm (\alpha + \beta)\,\%$$

Es $(7.0 \pm 2\,\%) / (5.0 \pm 1.0\,\%) =$

$= \frac{7.0}{5.0} \pm (2 + 1)\,\% = 1.4 \pm 3\,\%$

$$(x_0 \pm \alpha\,\%)^n = x_0^n \pm \alpha \cdot n\,\%$$

$$\sqrt[n]{x_0 \pm \alpha\,\%} = \sqrt[n]{x_0} \pm \frac{\alpha}{n}\,\%$$

$x = 4.0 \pm 0.4$ $\qquad$ $y = x - \sqrt{x}$

$y = 2 \pm ?$

$\sqrt{4.0 \pm 0.4} =$

$= \sqrt{4.0 \pm 10\,\%}$

$(4.0 \pm 0.4) - (2.0 \pm 0.1) =$ $\qquad$ $= 2.0 \pm 5\,\%$

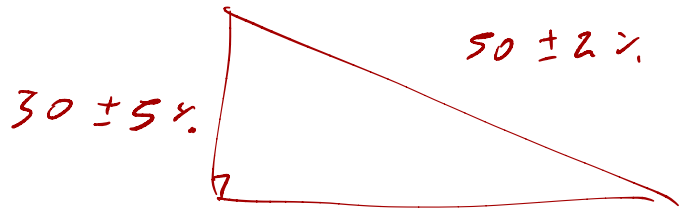
$= 2.0 \pm 0.5$?? $\qquad$ $= 2.0 \pm 0.1$

$x = 4.0 \pm 0.4$

$3.6 \leq x \leq 4.4$

$1.7 \leq y \leq 2.3$

$$y = 2.0 \pm 0.3$$



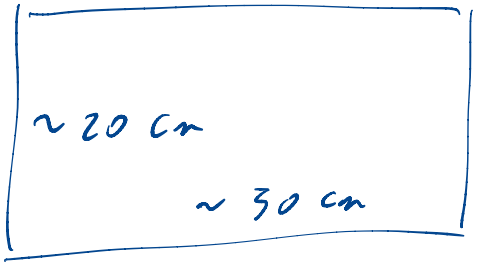
$$\sqrt{(50 \pm 2\%)^2 - (30 \pm 5\%)^2} =$$

$$\sqrt{(2500 \pm 4\%) - (900 \pm 10\%)} =$$

$$\sqrt{(2500 \pm 100) - (900 \pm 90)} =$$

$$\sqrt{1600 \pm 190} = \sqrt{1600 \pm 11.88\%} =$$

$$= 40 \pm 5.94\% \quad = 40 \pm 6\%$$



$A = ____ \pm 1\ cm^2$

$$A = (30\ cm \pm \delta) \cdot (20\ cm \pm \delta)$$

$$= \left(30\ cm \pm \frac{\delta}{30\ cm} \cdot 100\,\%\right) \cdot \left(20\ cm \pm \frac{\delta}{20\ cm} \cdot 100\,\%\right)$$

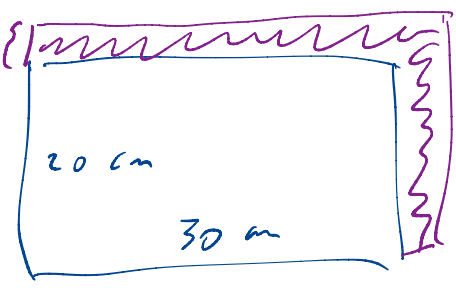
$$= 600\ cm^2 \pm \delta \left(\frac{1}{30\ cm} + \frac{1}{20\ cm}\right) \cdot 100\,\%$$

$$= 600\ cm^2 \pm \delta \left(\frac{1}{30\ cm} + \frac{1}{20\ cm}\right) \cdot 600\ cm^2$$

$$\delta \left(\frac{1}{30\ cm} + \frac{1}{20\ cm}\right) \cdot 600\ cm^2 = 1\ cm^2$$

$$\delta \cdot \frac{50}{cm} = 1 \qquad \delta = \frac{1}{50}\ cm = 0.2\ mm$$

0.2 mm



$0.2\ m \cdot 50\ cm$

$= 1\ cm^2$