Esponenziali e logaritmi

$a > 0$

$a^n = \underbrace{a \cdot a \cdot \ldots \cdot a}_{n \text{ volte}}$

$a^n \cdot a^m = a^{n+m}$

$a^n \cdot b^n = (a \cdot b)^n$

$a^{\frac{1}{2}} = \sqrt{a}$

$\underbrace{\sqrt{a} \cdot \sqrt{a}}_{a} = a^{\frac{1}{2}} \cdot a^{\frac{1}{2}} = a^{\frac{1}{2}+\frac{1}{2}} = a^1$

a

$a^{\pi} \quad a^{\sqrt{2}} \quad a^{-1.7} \quad \ldots$

$b^0 = 1$ $\quad b^x \cdot b^y = b^{x+y}$ $\quad (b^x)^y = b^{x \cdot y}$

$b^1 = b$ $\quad \frac{b^x}{b^y} = b^{x-y}$ $\quad \sqrt[n]{x} = x^{\frac{1}{n}}$

$1^x = 1$

$(a \cdot b)^x = a^x b^x$ $\quad \left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$

Es $\quad a^9 = 1000 \qquad a^6 = ?$

$a^3 = a^{\frac{9}{3}} = \left(a^9\right)^{\frac{1}{3}} = \left(10^3\right)^{\frac{1}{3}} = 10^{3 \cdot \frac{1}{3}} = 10^1 = 10$

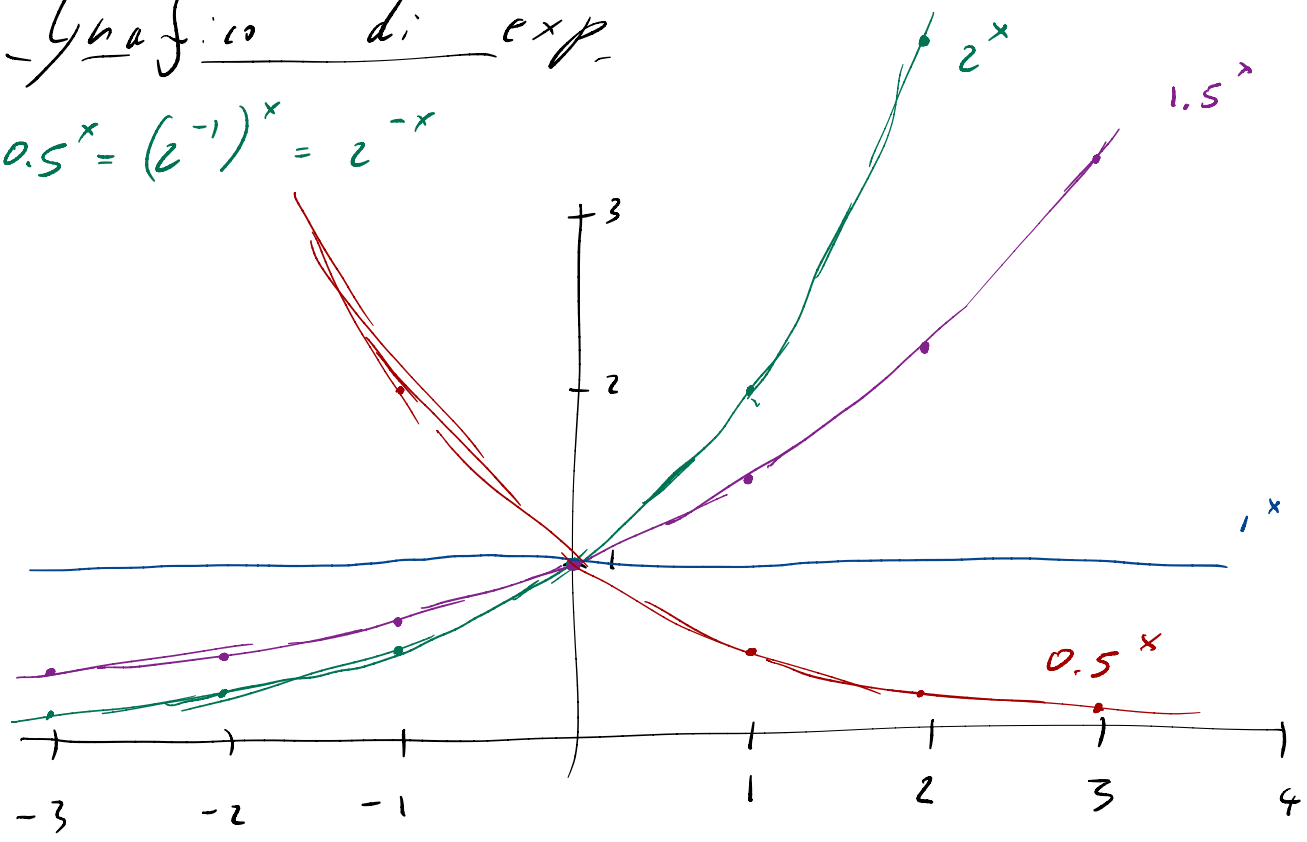
$a^6 = a^{3 \cdot 2} = \left(a^3\right)^2 = 10^2 = 100.$

$$\sqrt[3]{2.7 \cdot 10^7} = \left(2.7 \cdot 10 \cdot \frac{1}{10} \cdot 10^7\right)^{\frac{1}{3}} = 27^{\frac{1}{3}} \cdot \left(10^6\right)^{\frac{1}{3}}$$

$$= 3 \cdot 10^2 = 300$$

Grafico di exp

$0.5^x = \left(2^{-1}\right)^x = 2^{-x}$



$100^x > 10 \cdot 10^x$

$10^{2x} > 10^{1+x}$ $\leftarrow 10 > 1$

$2x > 1 + x$

$x > 1$

$0.01^x > 0.1 \cdot 0.1^x$

$0.1^{2x} > 0.1^{1+x}$ $\leftarrow 0.1 < 1$

$2x < 1 + x$

$x < 1$

Calcoliamo 10^x

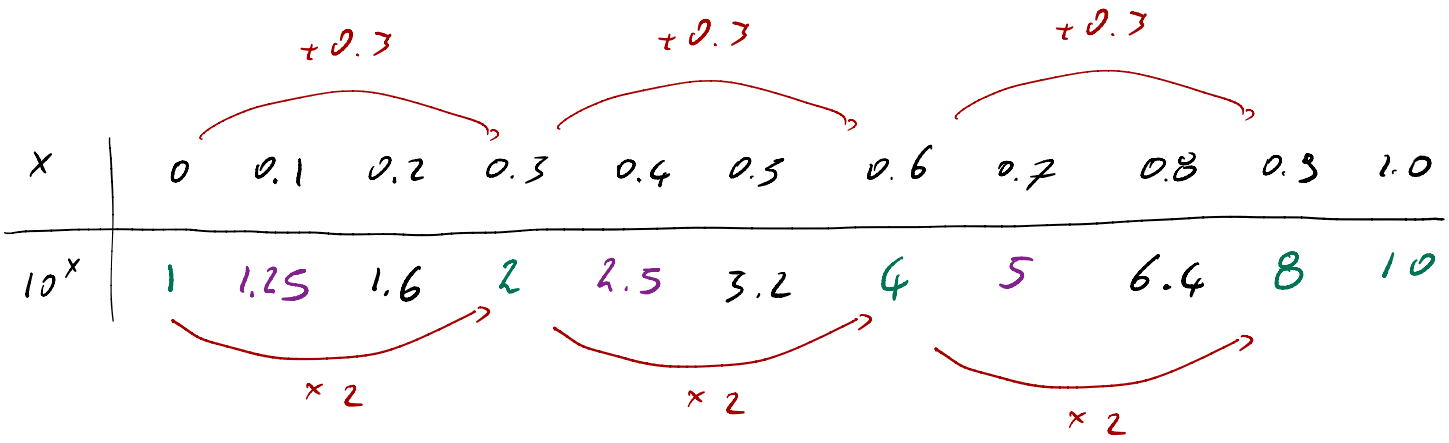
$10^{3.7} = ?$

$10^{3.7} = 10^{3+0.7} = 10^3 \cdot 10^{0.7} = 1000 \cdot 10^{0.7}$

$10^{0.3} \approx 2 \quad (1.99526\ldots)$

$10^{0.7} = 10^{1-0.3} = \frac{10^1}{10^{0.3}} \approx \frac{10}{2} = 5$

$10^{3.7} \approx 5000 \quad (\sim 5012)$



$10^{0.2} = 10^{0.3-0.1} = \frac{10^{0.3}}{10^{0.1}} = \frac{2}{1.25} = 1.6$

$e = 2.7182818\ldots$

e^x

ESERCIZIO B *dall'esame del 17·iii·2025, con una domanda in più*

Un condotto versa 1.50 ± 0.05 litri d'acqua al secondo in una vasca, inizialmente vuota, avente pianta quadrata di lato 4.00 ± 0.10 m e profondità 1.25 ± 0.05 m. Calcola il tempo impiegato per riempire la vasca in minuti, indicando l'errore assoluto. Lasciando invariati gli altri dati, con quale incertezza bisognerebbe misurare il lato della vasca affinché il tempo risulti determinato entro ± 20 minuti?

$$\text{tempo} = \frac{V}{(1.50 \pm 0.05)\frac{L}{s}} = \frac{[m^3]}{\frac{[m^3]}{[s]}} = [s]$$

0.03

$4.00 \pm 2.5\%$ $\qquad$ $1.25 \pm 4\%$

$$\Rightarrow \frac{(4.00 \pm 0.10)^2\, m^2 \; (1.25 \pm 0.05)\, m}{(1.50 \pm 0.05)\frac{(dm)^3}{s}}$$

0.83% $\qquad$ 1.50 ± 3.33 $\qquad$ 7.33

$$= \left(13.33 \pm \left(2.5\% \cdot 2 + 4\% + 3.33\%\right)\right) \frac{m^3}{\frac{(dm)^3}{s}}$$

$\pm 12.33\%$

$9\% - 7.33\% = 1.67\%$

$$\frac{1000}{60}\ min = 5 \cdot \frac{m^3}{(0.1\ m)^3} \cdot \frac{1\ min}{60\ s}$$

$0.1^3\ m^3$

$$= \left(13.33 \pm 12.33\%\right) \cdot \frac{1000}{60}\ min$$

$$= \left(\left(13.33 \cdot \frac{1000}{60} \right) \pm 12.33\,\% \right) \text{min}$$

± 27.4 → ± 9.00

$$= \left(222.17 \pm 12.33\,\% \right) \text{min}$$

± 30 → ± 20

$$= \left(220 \pm 30 \right) \text{min}$$

$$1 \frac{m^3}{\frac{(dm)^3}{s}} = x \text{ min}$$

$$1 \frac{m^3 \cdot s}{(dm)^3} = x \text{ min}$$

$$1 \frac{m^3 \cdot s}{(0.1 \cdot m)^3} = x \cdot 60 s$$

$$1 \frac{m^3}{0.1^3 m^3} = x \cdot 60$$

$$x = \frac{1}{0.1^3 \cdot 60} = \frac{1000}{60}$$

$2 \cdot \alpha + 4\% + 3.33\% = 9\%$

↓

$\alpha = 0.83\%$