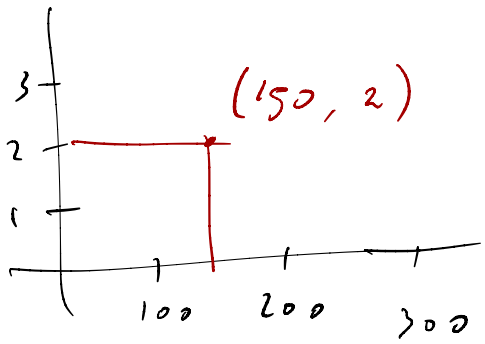
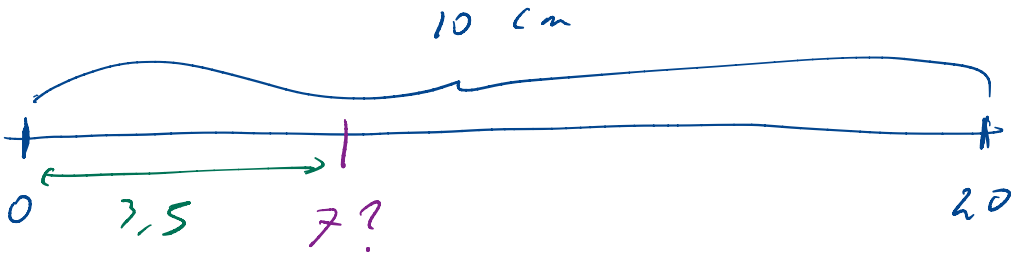
Scala logaritmiche



SCALA LINEARE

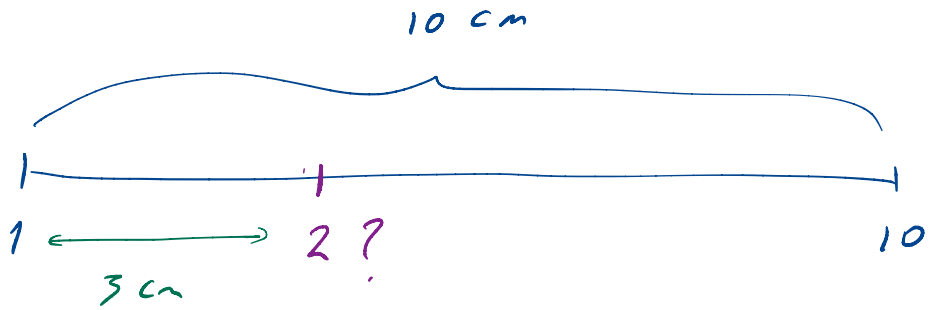
↓



$10\ cm : 20 - 0 = x : 7 - 0$

↓

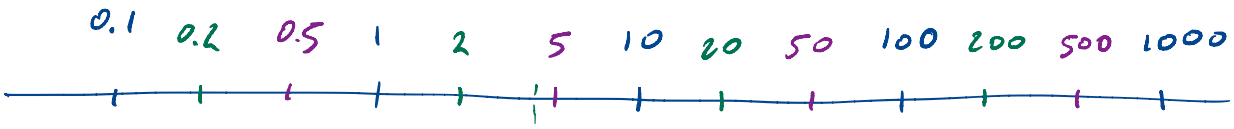
$x = 3.5\ cm$



$10\ cm : \underbrace{\log_{10} 10 - \log_{10} 1}_{1} = x : \underbrace{\log_{10} 2 - \log_{10} 1}_{0.3}$

↓

$x = 3\ cm$



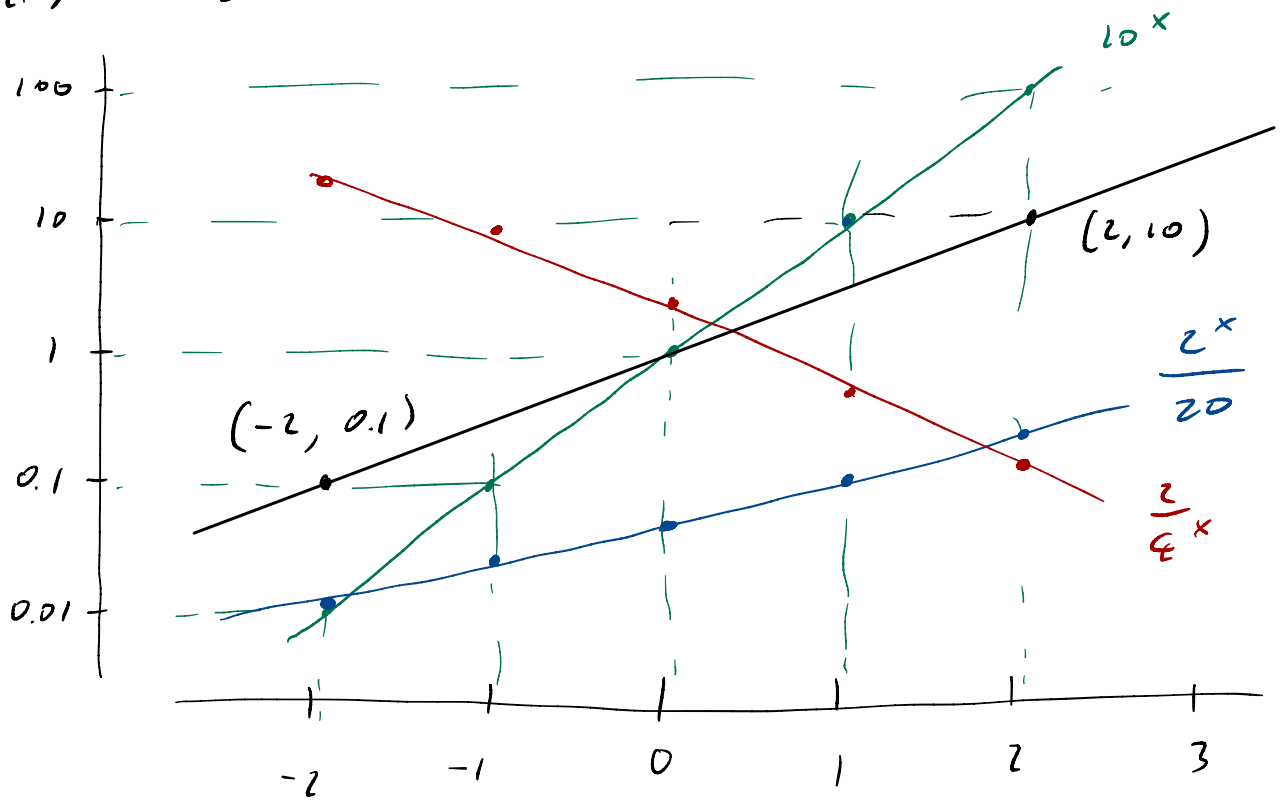
Grafici usuali : lin - lin

lin - log } semi-log.

log - lin }

log - log ← bi-log.

$f(x) = a \cdot b^x$



$$\begin{cases} a \cdot b^{-2} = 0.1 \\ a \cdot b^{2} = 10 \end{cases}$$

(-2, 0.1)

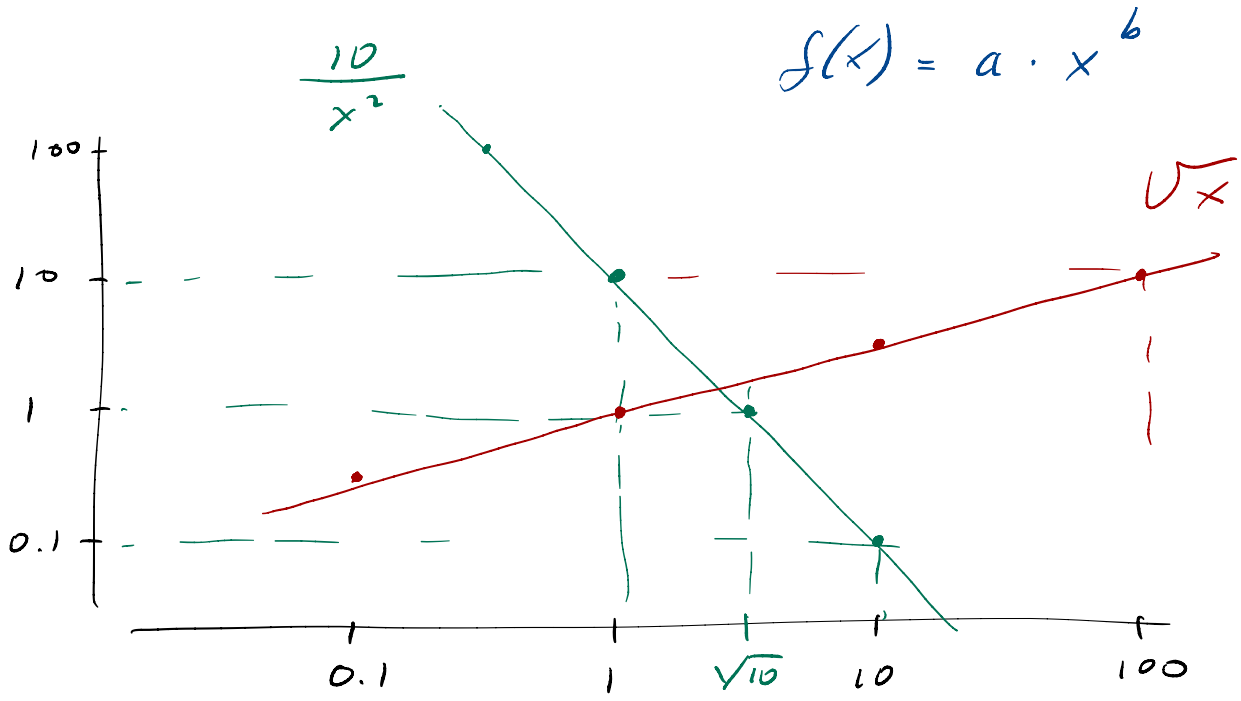
(2, 10)

$$\frac{0.1}{b^{-2}} \cdot b^2 = 10 \Rightarrow b^4 = 100 \Rightarrow b \simeq \sqrt[4]{100} \simeq 3.2$$

$$a = \frac{0.1}{b^{-2}}$$

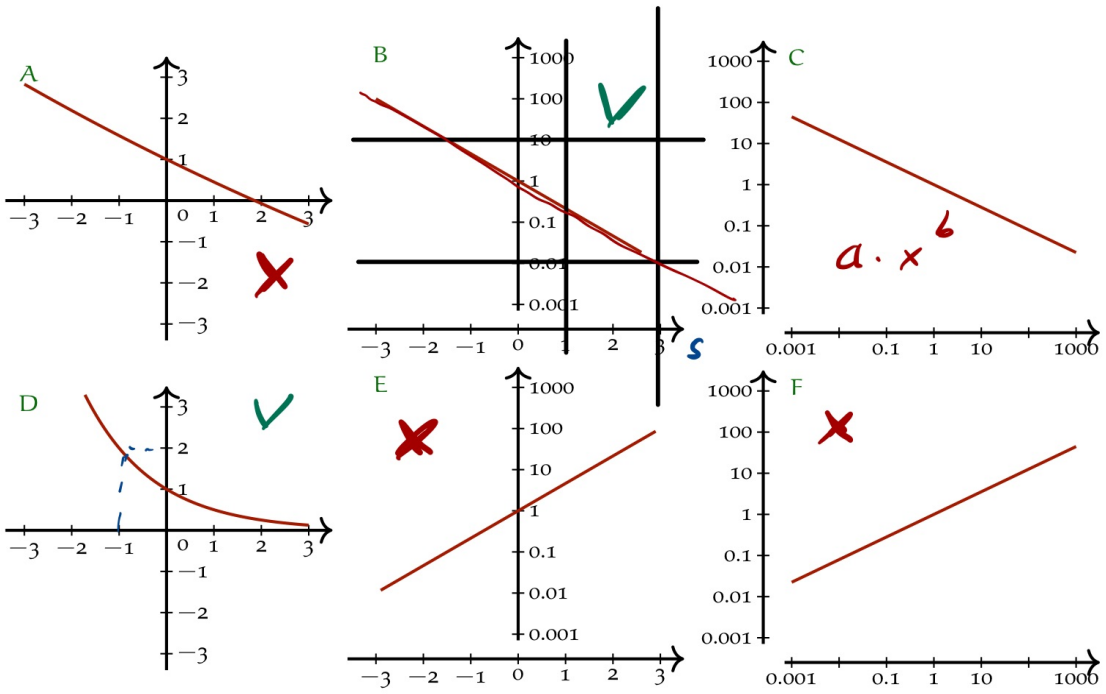
$a = 1$

$f(x) = 1 \cdot 3.2^x$


$f(x) = a \cdot x^b$
$\frac{10}{x^2}$
$\sqrt{x}$
100
10
1
0.1
0.1
1
$\sqrt{10}$
10
100

Esercizio 81 (soluzione a p. ??)

Determina quali dei grafici seguenti rappresentano un decadimento esponenziale, e, per ciascuno di questi, trova il tempo di dimezzamento. In tutti i grafici le ascisse sono indicate in secondi.



$0.5 \qquad 1$

$3.5 \qquad 0.01$

$$f(x) = a\left(\frac{1}{2}\right)^{\frac{t}{\tau}}$$

$$\begin{cases} 1 = a \cdot \left(\frac{1}{2}\right)^{\frac{0.5}{\tau}} \\ 0.01 = a \cdot \left(\frac{1}{2}\right)^{\frac{3.5}{\tau}} \end{cases} \Rightarrow \quad a = 1$$

$$0.01 = \left(\frac{1}{2}\right)^{\frac{3.5}{\tau}}$$

$$\frac{3.5}{\tau} = \log_{\frac{1}{2}} 0.01 = \frac{\log_{10} 0.01}{\log_{10} \frac{1}{2}} \approx \frac{+2}{+0.3}$$

$$\tau = 3.5 \cdot \frac{0.3}{2} = 0.45 \text{ s}$$

Esercizio 82 (soluzione a p. ??)
La concentrazione di un farmaco nel plasma di un paziente, dopo la somministrazione per via endovenosa, decresce esponenzialmente con un tempo di dimezzamento di 8.0 ore.

1. Quanto tempo occorre perché una concentrazione iniziale di 5.0 mg/L scenda a 0.10 mg/L?
2. A ogni somministrazione di una dose, la concentrazione plasmatica aumenta, molto rapidamente, di 5.0 mg/L, poi riprende a decrescere esponenzialmente. Il paziente riceve la prima dose al tempo t = 0, e poi una dose ogni 12.0 ore. Rappresenta con un diagramma cartesiano l'andamento della concentrazione plasmatica nelle 48 ore successive alla prima somministrazione.
3. A lungo andare, fra quali valori oscilla la concentrazione plasmatica del paziente?

$$\text{conc.}(t) = 5.0 \frac{\text{mg}}{\text{L}} \cdot \left(\frac{1}{2}\right)^{\frac{t}{8\text{h}}} = 0.10 \frac{\text{mg}}{\text{L}}$$

$$\frac{t}{8\text{h}} = \log_{\frac{1}{2}} \left(\frac{0.10}{5.0}\right) = \frac{\log_{10} 0.02}{\log_{10} 0.5} \approx \frac{-1.7}{-0.3}$$

0.02

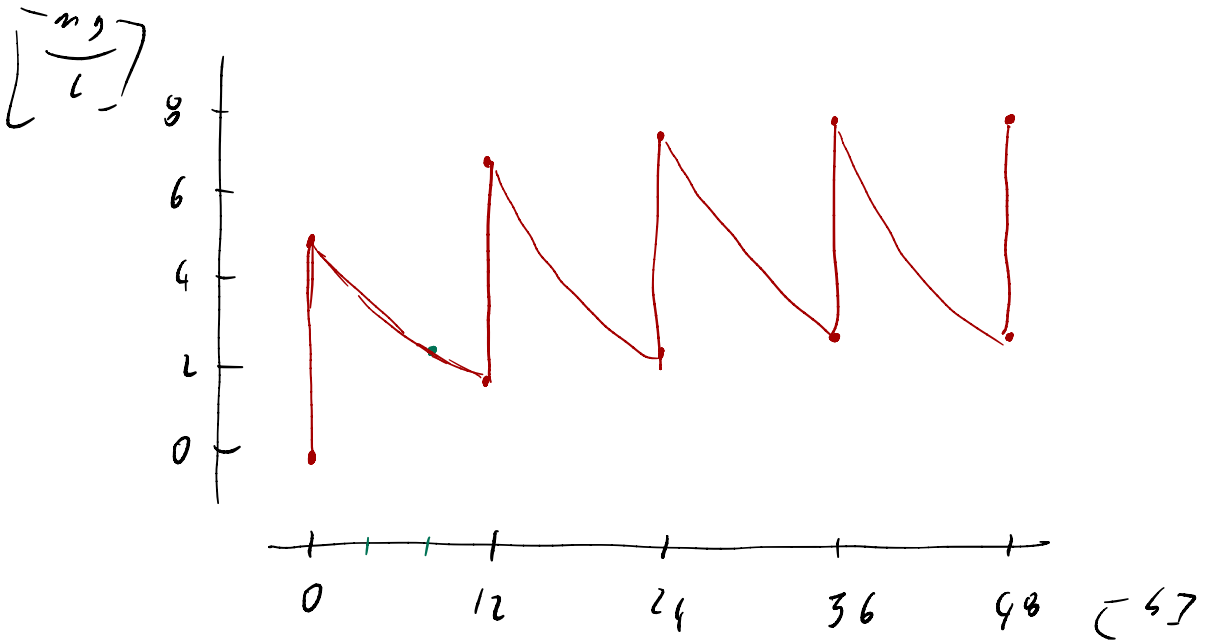
$$t = \frac{1.7}{0.3} \cdot 8\,\text{h} \approx 45\,\text{h}$$

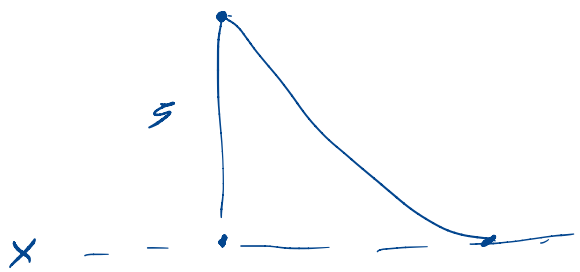
②

0 h	12 h	24 h	36 h	48 h
0	1.77	2.39	2.61	2.69
5	6.77	7.39	7.61	7.69

$\left[\frac{mg}{l}\right]$

$5 \cdot \left(\frac{1}{2}\right)^{\frac{12\,h}{8\,h}}$ $\quad 6.77 \left(\frac{1}{2}\right)^{\frac{12\,h}{8\,h}}$





$\left(x + 5\,\frac{mg}{l}\right) \cdot \left(\frac{1}{2}\right)^{\frac{12\,h}{8\,h}} = x$

$$\left(x + 5\,\frac{mg}{L}\right) \cdot \left(\frac{1}{2}\right)^{\frac{12\,\text{J}}{8\,\text{J}}} = x \qquad 1.5$$

$$x + 5\,\frac{mg}{L} = x \cdot 2^{1.5}$$

$$x = 5\,\frac{mg}{L} \cdot \frac{1}{2^{1.5} - 1} \approx 2.73\,\frac{mg}{L}$$

$$2.73\,\frac{mg}{L}$$