$$\frac{dy}{dx} \rightsquigarrow \forall \varepsilon > 0 \ \exists \delta \ldots$$

Prob. Data

$$S(x) = c_0 + \sum_{i=1}^{\infty} a_0 \sin(ix) + b_0 \cos(ix)$$

Supponiamo che $\forall x \in \mathbb{R}$ $S(x)$ converge a 0

E' vero che $c_0 = a_1 = a_2 = \ldots = b_1 = \ldots = 0$?

Cantor (1870) : Sì.

Def $X \subseteq \mathbb{R}$ è di unicità se

$$\forall x \notin X \ S(x) = 0 \implies c_0 = a_1 = \ldots = b_1 = \ldots = 0$$

Fatto $X \subseteq \mathbb{R}$ è di unicità se ha $(*)$ $\quad f(x) = \alpha x + \beta$

$(*)$ Data $f : \mathbb{R} \to \mathbb{R}$ continua se

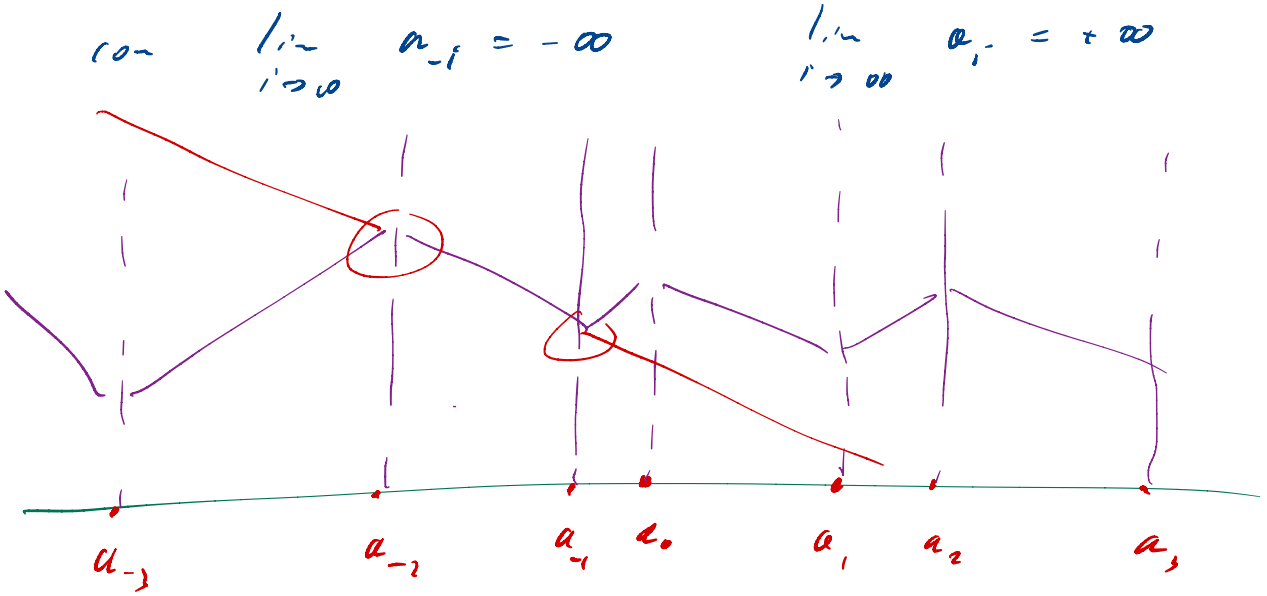
- $\forall \,]a,b[$ con $]a,b[\cap X = \emptyset$ $\quad f|_{]a,b[}$ è lineare
- $\forall x$ se f ha derivate dx e sx in x allora queste coincidono.

"non ha p.ti angolosi"

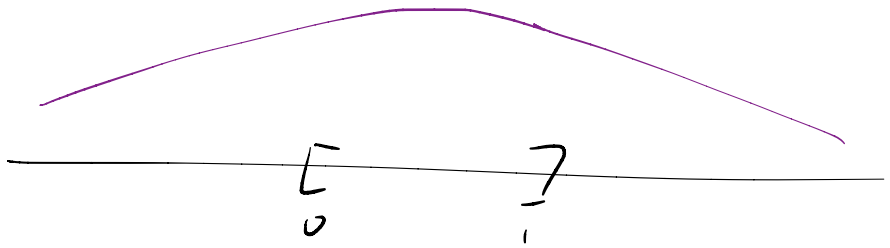
$\to$ f è lineare.

Ex. $X = \{ \dots a_{-2}, a_{-1}, a_0, a_1, a_2 \dots \}$

$a_{-2} < a_{-1} < a_0 < a_1 < a_2 < \dots$

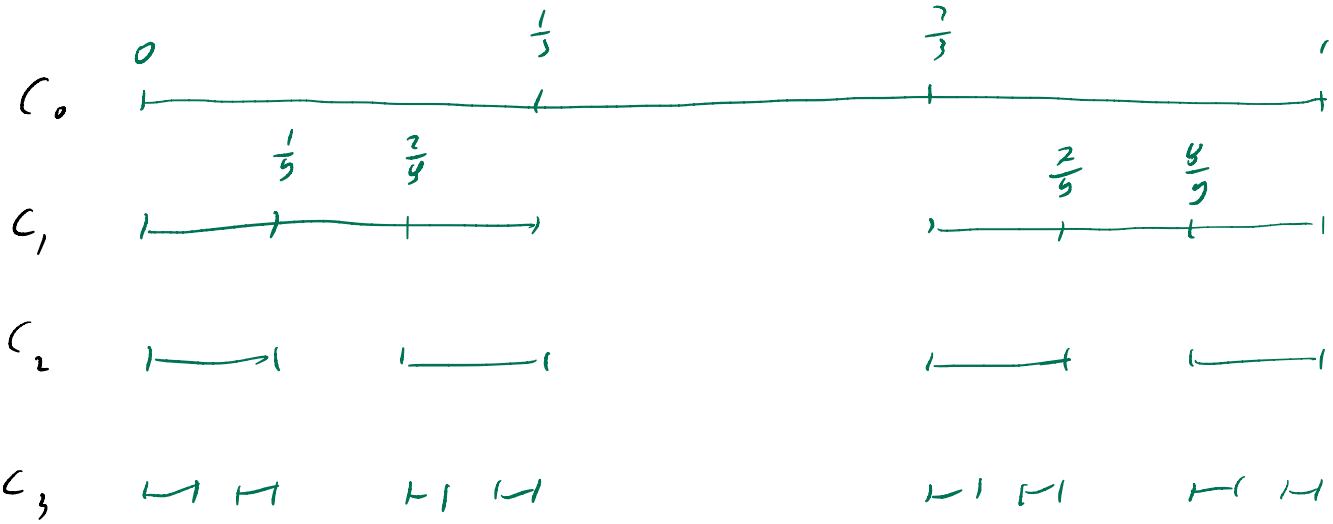


Non cs: $[0,1]$



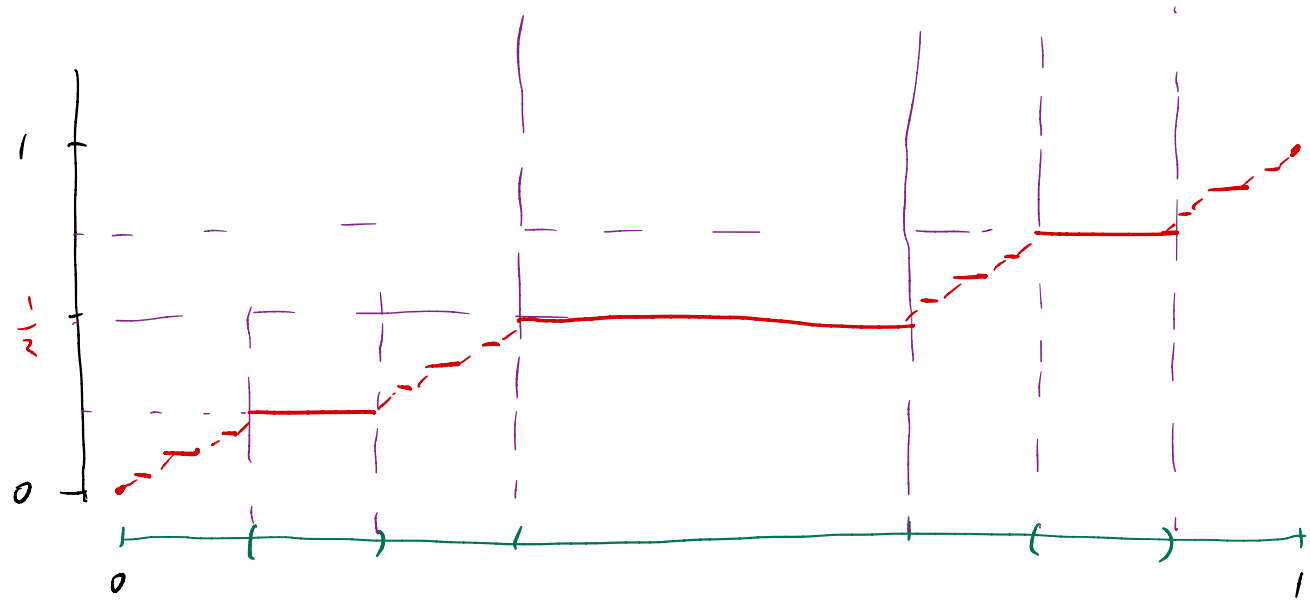
Q.

Mon esempio (ins. di Cantor)



$\bigcap$ interseco

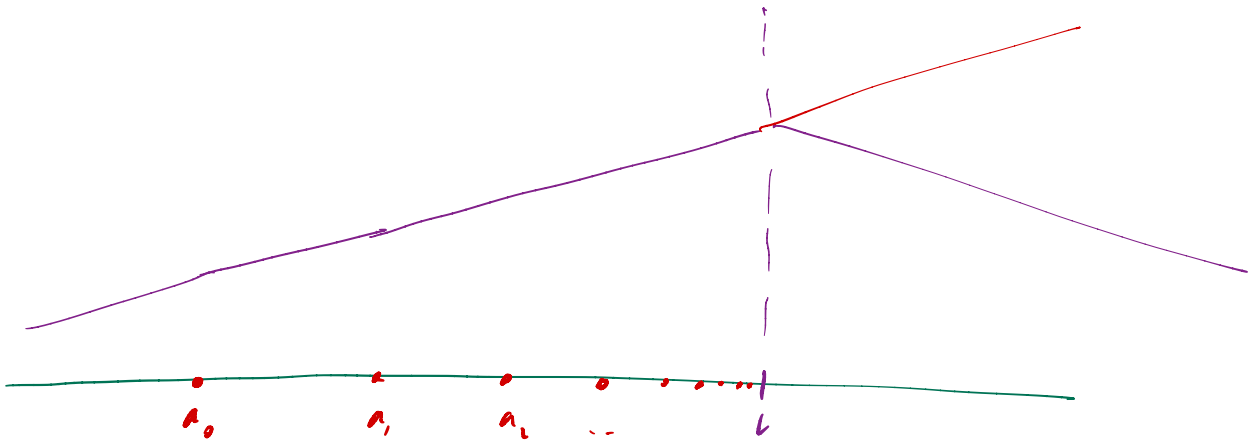


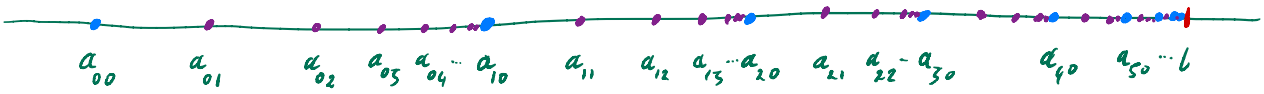


Esempio



$X = \{a_0, a_1, a_2, \ldots, L\}$ $\quad a_i \uparrow L$

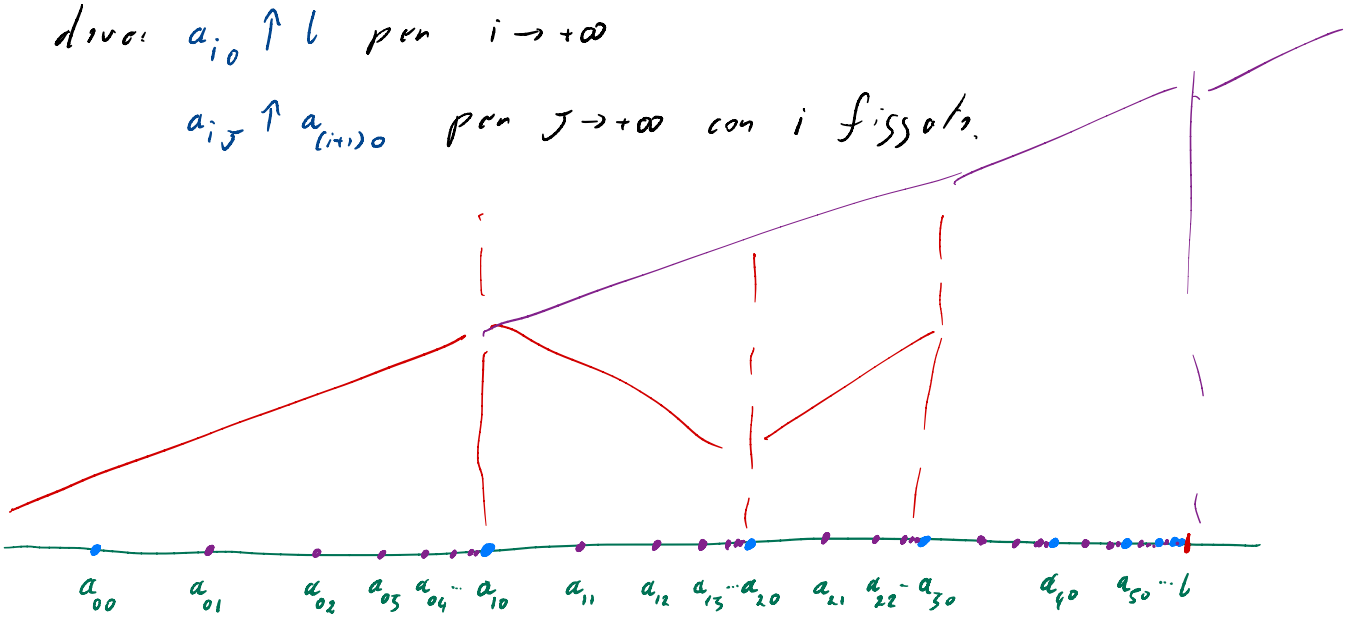




$$X = \{ a_{ij} \mid i, j \in \mathbb{N} \} \cup \{ l \}$$

dove $a_{i0} \uparrow l$ per $i \to +\infty$

$a_{ij} \uparrow a_{(i+1)0}$ per $j \to +\infty$ con i fissato.



Def Un insieme X è numerabile se

$$X = \{ a_i \mid i \in \mathbb{N} \}$$

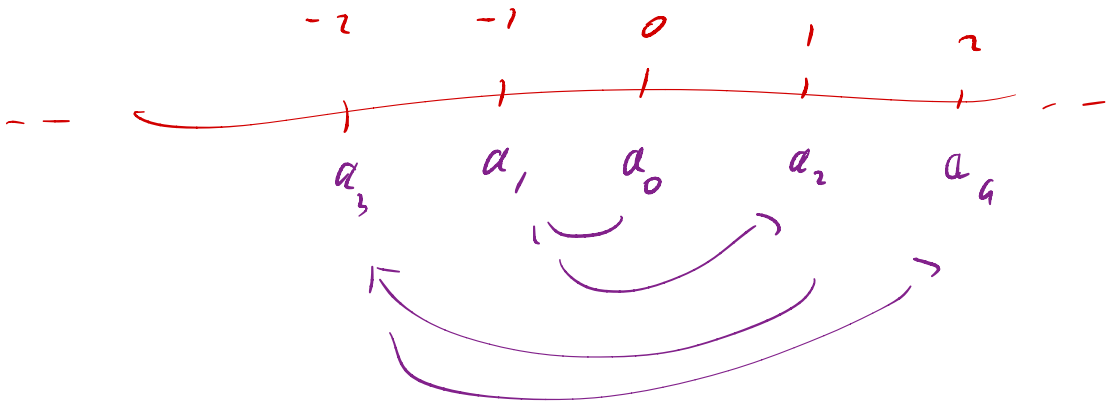
per opportuni a_i tutti diversi fra loro.

$\mathbb{N}$ $\qquad a_i = i$

$\{$n. dispari$\}$ $\qquad a_i = 2i+1$

$\{$n. primi$\}$ $\qquad a_i =$ "i-esimo primo"

$\mathbb{Z}$ $\qquad a_i = (-1)^i \left\lceil \frac{i}{2} \right\rceil$



$\mathbb{N} \times \mathbb{N} = \{ (a,b) \mid a,b \in \mathbb{N} \}$

$$f : \mathbb{N} \times \mathbb{N} \longrightarrow \mathbb{N}$$

$$(x,y) \longmapsto 2^x (1+2\cdot y) - 1$$

$$z = 2^x (1+2y) - 1 \quad \longleftrightarrow \quad z+1 = 2^x (1+2y)$$

Per enumerare $\mathbb{N} \times \mathbb{N}$ prendo

$$a_i = f^{-1}(i)$$

Prop. X è num., $Y \subseteq X$ infinito $\Rightarrow$ Y è num.

Dim $X = \{a_0, a_1, a_2, \dots\}$

Considero $b_j = a_{i_j}$ sottosucc. degli elem. che $\in Y$.

Più precisamente:

$i_0 = \min\{i \mid a_i \in Y\}$ $\quad i_{j+1} = \min\{i > i_j \mid a_i \in Y\}$

Prop. X, Y num. $\Rightarrow$ $X \times Y$ è num.

$\{(a,b) \mid a \in X, b \in Y\}$

Dim $X = \{a_i \mid i \in \mathbb{N}\}$

$Y = \{b_j \mid j \in \mathbb{N}\}$

$\mathbb{N} \times \mathbb{N} = \{(i_t, j_t) \mid t \in \mathbb{N}\}$

$\downarrow$

$X \times Y = \{(a_{i_t}, b_{j_t}) \mid t \in \mathbb{N}\}$

Esempio $\mathbb{Q}$ è numerabile

$$\mathbb{Q} = \{ (num., den.) \mid \begin{matrix} num \in \mathbb{Z} \\ den \in \mathbb{N} \setminus \{0\} \end{matrix} \quad MCD(num., den.) = 1 \}$$

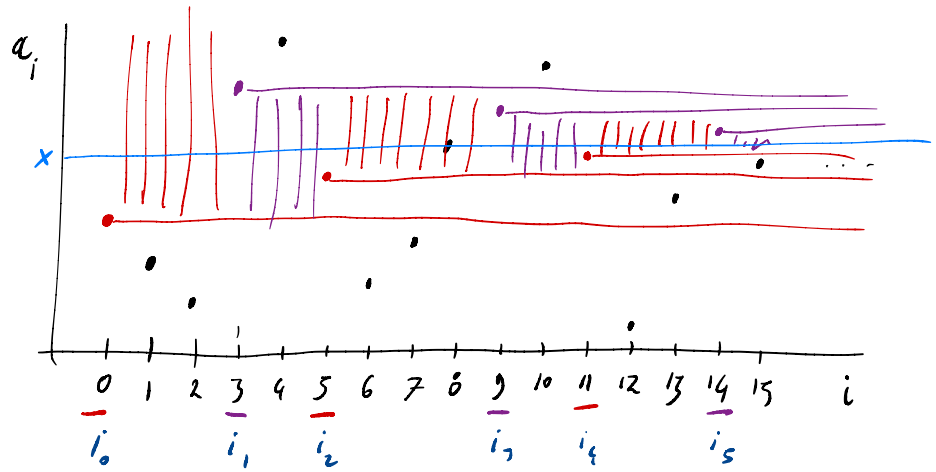
$$\subseteq$$

$$\mathbb{Z} \times \mathbb{N}$$

Non esempio $\mathbb{R}$

Per assurdo $\mathbb{R} = \{ a_0, a_1, a_2, \dots \}$

Scomodo trovare $x \in \mathbb{R}$ t.c. $\forall i \quad x \neq a_i$



$a_{i_{2N}} \uparrow l_p$ $\qquad$ $a_{i_{2N+1}} \downarrow l_d$

$$l_p \leq x \leq l_d$$

Esercizio C (l'insieme di Cantor) non è num.

Teo (Cantor – Lebesgue)

$X \subseteq \mathbb{R}$ chiuso e numerabile $\Rightarrow$ X ha (*)

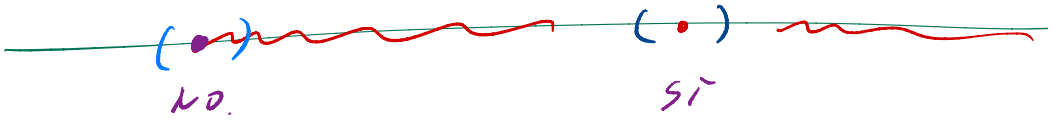
($\Leftrightarrow$ è di unicità).

Def $X \subseteq \mathbb{R}$

$X' = X \setminus \{\text{punti isolati di } X\}$

$\Uparrow$

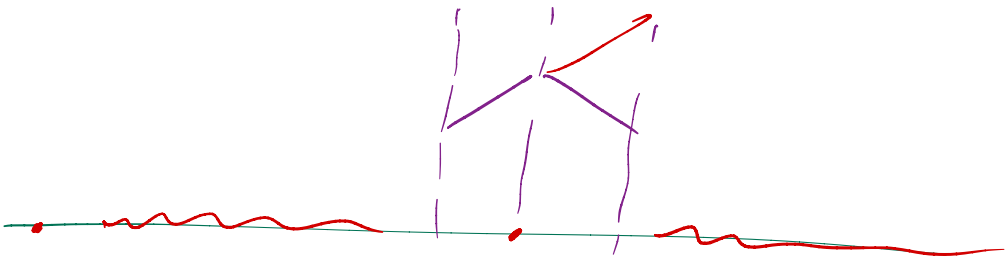
$x \in X$ t.c. $\exists \varepsilon > 0 \quad]x-\varepsilon, x+\varepsilon[\cap X = \{x\}$

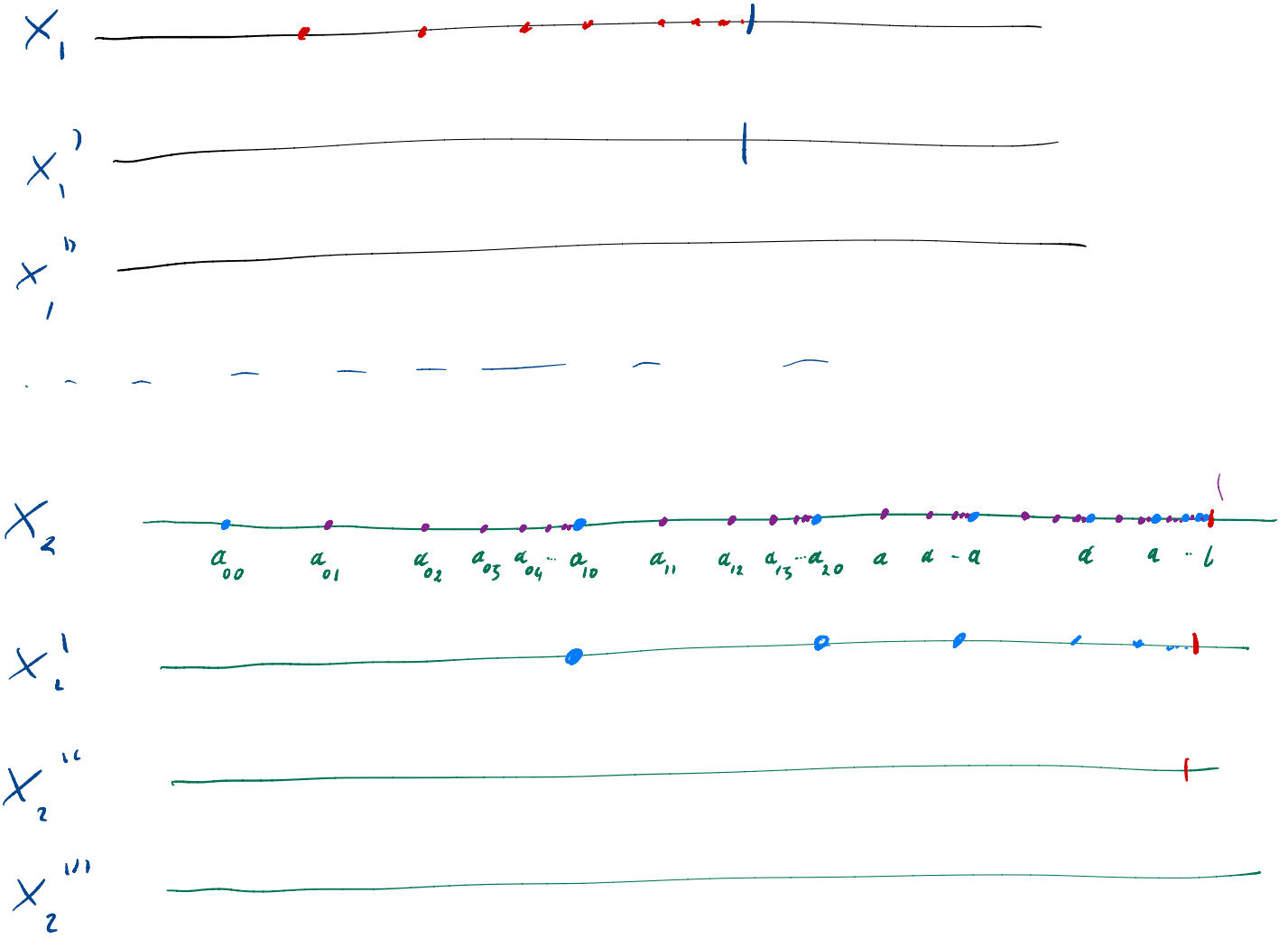


Lemma Se X è chiuso e X' ha (*)

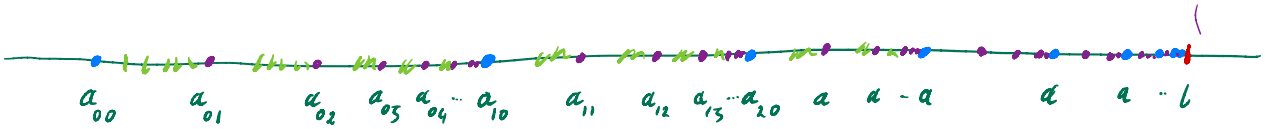
allora X ha (*)

"Dim"





X_3 succ. di succ. di succ.



X_n $\quad X_n^{(n)} \neq \emptyset \quad X_n^{(n+1)} = \emptyset$

