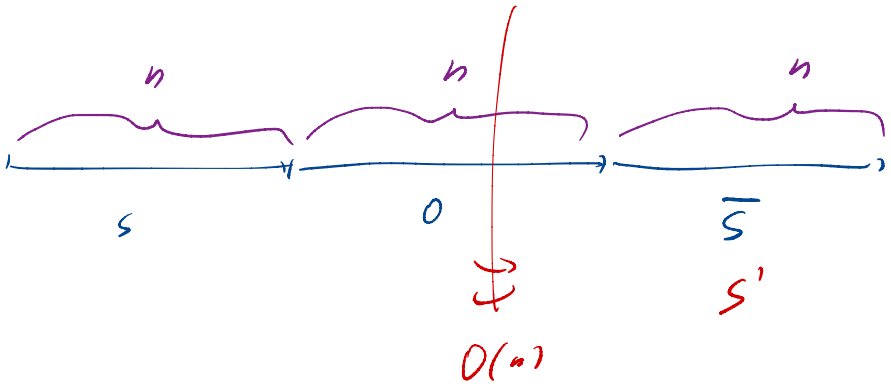


Def Sia $f : \mathbb{N}^n \to \mathbb{N}_\perp$ è Turing-computabile se esiste una T.M. $M = (\Sigma, Q, \delta)$ t.c.

$0, 1, \triangleright \in \Sigma \quad 0, 1 \in Q \quad \delta(0, x) = (0, x, \downarrow)$

se $f(x_1 \dots x_n) = y \neq \perp$ $\quad input(x_1 \dots x_n) \xrightarrow{M^*} output(y)$

se $f(x_1 \dots x_n) = \perp$ $\quad input(x_1 \dots x_n) \xrightarrow{M^*} (\neq 0, -, -)$

$input(x_1 \dots x_n) = (1, 0, \triangleright \underbrace{11\dots1}_{x_1} 0 \underbrace{11\dots1}_{x_2} 0 \dots \underbrace{11\dots1}_{x_n} 00\dots)$

$output(y) = (0, 0, \triangleright \underbrace{111\dots11}_{y} 000\dots)$

Teo f ricorsiva $\to$ f è T-comp.

Teo Esiste una f. ricorsiva $U : \mathbb{N}_\perp^2 \to \mathbb{N}_\perp$ tale che se f è Turing-computabile allora esiste $\ulcorner f \urcorner \in \mathbb{N}$ t.c.

$\forall x_1 \dots x_n \quad f(x_1 \dots x_n) = U(\ulcorner f \urcorner, \langle x_1 \dots x_n \rangle)$

Con Turing-comp. $\Longrightarrow$ ricorsivo

Dim

$x_1 \dots x_n \longmapsto U(\ulcorner f \urcorner, \langle x_1 \dots x_n \rangle)$

$\cons(x_1, \cons(x_2, \dots))$

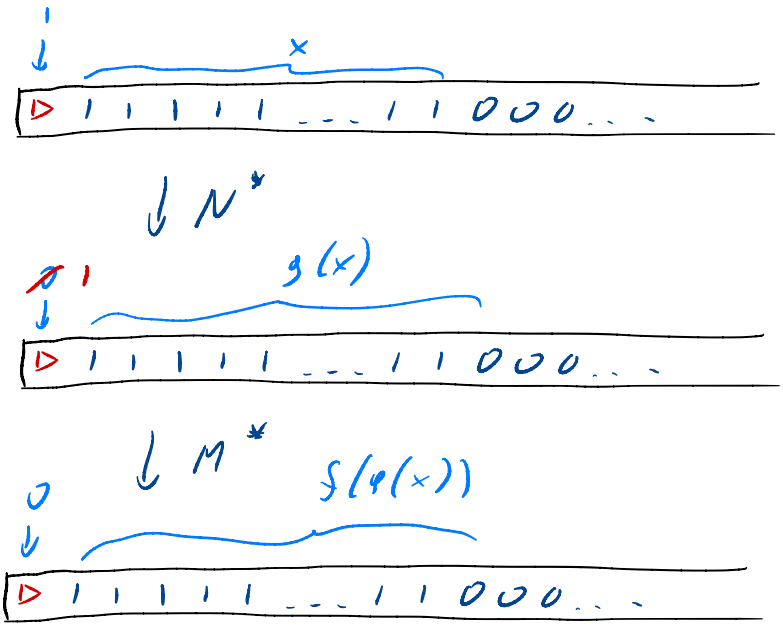
è ricorsiva

Teo f ricorsiva $\rightarrow$ f è T-comp.

Dim (idea)

caso particolare

$x \longmapsto f(g(x))$

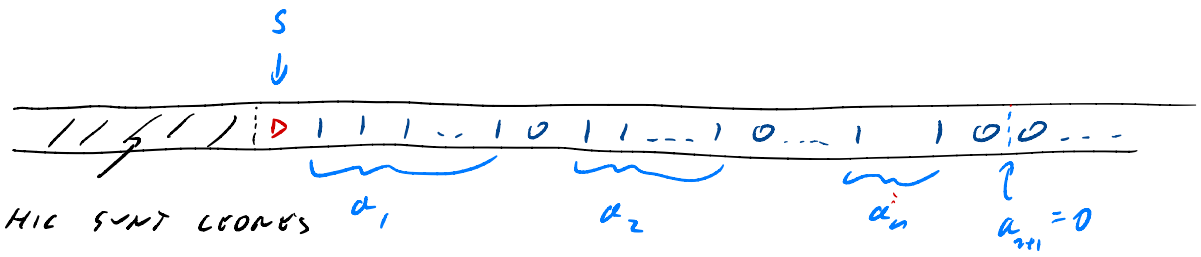


$Q = Q_M \cup Q_N$

$\Sigma = \Sigma_M \cup \Sigma_N$

$$\delta(q,x) = \begin{cases} \delta_N(q,x) & q \in Q_N - \{0_N\} \\ \delta_M(1_M,x) & q = 0_N \\ \delta_M(q,x) & q \in Q_M \end{cases}$$

$1 := 1_N \qquad 0 := 0_M$



$$s \triangleright a_1 \dots a_n = s \triangleright a_1 \dots a_n 0 0 0$$

$$s \to push_{n,i} \to s' \quad (s \triangleright a_1 \dots a_n) = s' \triangleright a_1 \dots a_n a_i$$

$$s \to rem_{n,i} \to s' \quad (s \triangleright a_1 \dots a_n) = s' \triangleright a_1 \dots a_{i-1} \; a_{i+1} \dots a_n$$

$$s \to exec_{n,k} f \to s' \quad (s \triangleright a_1 \dots a_n) = s' \triangleright a_1 \dots a_{n-k} \; f(a_{n-k+1} \dots a_n)$$

$$s \to \text{if } a_i = 0 \begin{cases} \nearrow s' \\ \searrow s'' \end{cases} \quad (s \triangleright a_1 \dots a_n) = \begin{cases} s' \triangleright a_1 \dots a_n & \text{se } a_i = 0 \\ s'' \triangleright a_1 \dots a_n & \text{se } a_i \neq 0 \end{cases}$$

$+1 \qquad \div 1 \qquad 0$

- composizione

$$(x,y) \mapsto f(g_1(x,y), g_2(x,y)) \qquad M\ N_1\ N_2$$

$1\ D\ x\ y$

$1 \to push_{2,1} \to 2$

$2\ D\ x\ y\ x$

$2 \to push_{3,2} \to 3$

$3\ D\ x\ y\ x\ y$

$3 \to exec_{4,2}\ N_1 \to 4$

$4\ D\ x\ y\ g_1(x,y)$

$4 \to push_{3,1} \to push_{4,2} \to exec_{5,2}\ N_2 \to 5$

$5\ D\ x\ y\ g_1(x,y)\ g_2(x,y)$

$5 \to rem_{4,1} \to rem_{5,1} \to exec_{2,2}\ M \to 0$

- n.c. primitiva

$$f(x_1, \dots x_n, 0) = \underline{h}(x_1, \dots x_n)$$

$$f(x_1, \dots x_n, y+1) = \underline{g}(x_1, \dots x_n, y, f(x_1, \dots x_n, y))$$

$$f(x_1, \dots x_n, \bot) = \bot$$

$1 \triangleright x\ y$

$1 \to push_{3,1} \to 2$

$2 \triangleright x\ y\ 0\ x$

$2 \to exec_{4,1}\ h \to 3$

$3 \triangleright x\ y \quad 0 \quad f(x,0) \quad (= h(x))$

$3 \triangleright x\ y-t\ t\ f(x,t) \qquad t = 0 \dots y$

$3 \to if\ a_2 = 0 \to rem_{4,1} \to rem_{3,1} \to rem_{2,1} \to 0$

$\qquad\qquad\quad \to 4$

$4 \to push_{4,2} \to exec_{5,1}\ \dot{-}1 \to push_{5,3} \to exec_{6,1}\ +1 \to 5$

$5 \triangleright x, y-t, t, f(x,t), y-t-1, t+1$

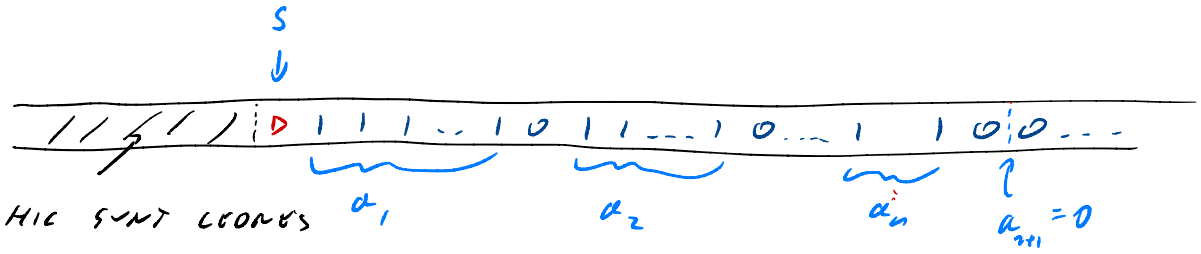
$5 \to push_{6,1} \to push_{7,3} \to push_{8,4} \to exec_{9,3}\ g \to 6$

$\delta(x, y-1, t, f(x,t), y-1-1, t+1, g(x, t, f(x,t)))$

$\delta(x, y-1, t, f(x,t), y-1-1, t+1, f(x, t+1))$

$\delta \to rem_{7,2} \to rem_{6,2} \to rem_{5,2} \to 3$

- minimization : ESCLAVO.



$$s \triangleright a_1 \dots a_n = s \triangleright a_1 \dots a_n 0 0 0$$

$$s \to \underset{n,k}{exec} f \to s' \quad (s \triangleright a_1 \dots a_n) = s' \triangleright a_1 \dots a_{n-k} f(a_{n-k+1} \dots a_n)$$

s

$\triangleright \; a_1 \; 0 \; a_2 \; 0 \; \cdots \; 0 \; a_{n-k+1} \; 0 \; \cdots \; a_n \; 0 \; 0 \; \dots$

α_1

$\triangleright \; a_1 \; 0 \; a_2 \; 0 \; \cdots \; 0 \; a_{n-k+1} \; 0 \; \cdots \; a_n \; 0 \; 0 \; \dots$

1_s

$\triangleright \; a_1 \; 0 \; a_2 \; 0 \; \cdots \; \triangleright \; a_{n-k+1} \; 0 \; \cdots \; a_n \; 0 \; 0 \; \dots$

0_s $\qquad \delta(0_s, \triangleright) = (\beta, 0, \leftarrow)$

$\triangleright \; a_1 \; 0 \; a_2 \; 0 \; \cdots \; \triangleright \; f(a_{n-k+1} \dots a_n) \; 0 \; 0 \; \dots$

β

$\triangleright \; a_1 \; 0 \; a_2 \; 0 \; \cdots \; 0 \; f(a_{n-k+1} \dots a_n) \; 0 \; 0 \; \dots$

$$\delta(\beta, x) = (\beta, x, \leftarrow)$$

$$\delta(\beta, \triangleright) = (s', \triangleright, \downarrow)$$